1 - 1 Introduction

In an effort to compete with film and TV, theatrical stage scenery has been growing larger, more complicated and more ambitions year after year. This trend began with Broadway shows such as Les Misérables and The Phantom of the Opera and continues today. This trend has been expanding from the commercial markets to regional theatres across the country. In order to meet the needs of these large scale and often non-traditional physical productions technical directors and theatre technicians need to understand the forces at work on structural members and be able to determine what is necessary to resist those forces. In the past scenery would often constructed using tried and true methods, or engineered using the “trial and error” or the “that should hold it up…” method. This is becoming less and less practical, economical and safe.

In the world of scenic construction there are certain things that are more or less standard. For example, platforms built from lumber will usually use either 2 x 4 or 1 x 6 in the construction and covered using ¾” plywood. You probably would then attach six legs of some sort, two on each end and two centered along the platform’s long side. Why do you build it this way? Because that is how you were taught to build platforms. It works. It always has and it always will. But what if the platform could only be two inches thick? What if it also had to span ten feet without support in the middle? What if it is a production of The Fat Boys Learn to Dance and all six of them do a kick line on the platform for the finale?! You could tell the designer and director that their artistic vision is impossible and there must be legs in the center or the platform must be thicker or you could calculate what materials you would need to use to make this artistic moment a reality.

Sarcasm aside, there should be no doubt that a technical director, scenic carpenter or theatrical technician must have a good working knowledge of simple structural mechanics. There are many books on the subject of engineering but most of them are directed at students of Engineering, not the theatrical technician. It is the author’s attempt here to compress a very lengthy and detailed subject into a simplified and manageable form, useful to the theatrical technician. This is not an easy task and involves simplifying or, in some cases, omitting much of the information. For example, it is rare that we will need to account for snow loads or have to account for seismic occurrences when designing indoor scenery, therefore this text will not discuss such matters.

Many professional engineers would resolutely object to such a text as this and they do have a point. This text cannot even begin to cover the actual complexities involved in even apparently simple structural cases. Since a chain is only as strong as its weakest link, even a slight mathematical error or error in logic could produce disastrous results. They feel that a little knowledge is a dangerous thing. What must be remembered at all times is that mastering the material covered in this book does not make one a structural engineer. That would require many years of school, concentrating solely on such matters. What mastering this material does give is a tool to provide reassurance that one’s methods and materials are sufficiently strong so as to provide for safe scenery. Anyone experienced in scenic construction already has a feel for the forces at work in a structure, but these mathematical
tools give some substance to your “gut reaction”. One should never throw away standard scenic construction methods simply because your numbers say there is sufficient strength in one’s new choice of material.

As the theatrical world expands into more and more elaborate construction it is becoming necessary to be able to dissect a particular system to determine what forces are at work and how those forces will affect the material the scenery is constructed from. Anyone who has seen a Broadway musical in the past 15 years can see this.

It is not necessary to have previously mastered physics, calculus or any higher mathematics to make use of the processes included in this text, although most of the formulas and expressions used have been derived using calculus. All that is necessary is knowledge of high school algebra and trigonometry and a scientific calculator.

Reading and mastering a text such as this does not make you an engineer. A structure designed to meet all parameters set down in this book does not guarantee the structure will not fail. A professional engineer should be consulted when dealing with large spans, large forces and/or a permanent installation where the structure will be subject to decay, abuse or misuse.

1 - 2 Definition of Terms

The branch of physics which studies the actions of forces on materials is called mechanics. A force can be thought of as a push or a pull on an object. For example, if you tie a rope to a hook in a wall and pull on the rope, you are said to be applying a force to the rope (later we will see how this force is also applied to the hook, the wall, etc). If the rope is strong enough to not break (fail) it is said to be resisting the force applied to it. A force could also be applied to a moving object such as a wrench falling from a grid. In this case gravity is the force pulling the wrench toward the center of the earth. As you let go of the wrench it starts out not moving at all, but then gravity pulls it down. This is called acceleration. A platform resting on six 2x4 legs on a stage floor is also being pulled on by gravity. The stage floor provides an equal and opposite force pushing back up on the platform legs and hence the platform itself. Statics is the branch of mechanics which studies bodies held motionless by balanced forces such as this and is known as static equilibrium. Before you let go of the wrench, your hand is supplying a force equal and opposite to the gravitational force pulling the wrench down. Gravity is pulling the wrench down and your hand is pushing the wrench up. In the rope example, if the rope is not tied to a wall, but is being held by a nearby electrician and you each pull equally as hard, neither of you will move. You are balanced or in static equilibrium. Dynamics on the other hand, is the study of bodies in motion and forces which are dependent upon time. Examples are the falling wrench or a really weak electrician pulling the rope. Most of our study will be contained within the field of Statics.
When an external force is applied to an object, the object produces forces to counter the external force. The molecules of the material like to be in the arrangement they are in (say a steel tube). If you try to pull them further apart or push them closer together, they will try to resist it. They don’t want to move! They like it where they are! These internal forces produce stress in the object. This isn’t the kind of stress you might feel before a test. It is a measurement of the ability of a material to resist external forces. For example, imagine you are a hanging speaker cluster from a cable connected to the roof steel of a coliseum. As you load speakers the force of gravity will be pulling the speaker down (an external force, gravity pulling the speaker and hence the cable). The cable will react by producing stress to exactly counter the force of gravity pulling the speaker. The molecules making up the steel cable want to stay together. If you continue loading speakers on this cable at some point the cable will not be capable of producing any more resisting force (stress) and it will break. If the object is not capable of producing sufficient force to counteract the external force the object is said to have failed. The force of gravity is still pulling the speaker down, but the cable is no longer able to pull against gravity. This will produce motion in the object (in this example falling) which is then studied using tools of dynamics. Even without studying the dynamics of this situation, we all intuitively know what will happen. The speaker will accelerate down toward the floor. When it hits the floor it will rapidly decelerate (and smash!). The floor now has an external force being applied to it (gravity is still pulling the speaker down). If the floor is capable of producing sufficient stress to resist this external force the speaker will stop. If not, then the floor will fail and the speaker will continue falling (i.e. it will crash through the floor). Strength of Materials is the study of the properties of materials which produce those internal forces necessary to resist external force. In other words, Strength of Materials study will allow us to determine if a material (such as the cable above) is capable of resisting the external force (the weight of the speaker). Statics, dynamics and strength of materials, combined together usually form what is known as structural mechanics or structural analysis (Simplified Engineering for Architects and Builders).

1 - 3 Units of measurement

There are two commonly used systems of measurement in the world today. One is usually called “the English system” (more accurately called the “U.S. Customary System” or sometimes the “Imperial” system) and the other is the Metric system or SI (for Systeme International). A complete switch over to SI in the United States seems inevitable, but it would mean changing many standards in the construction industry. So for the moment we must make due with the clunky English system. Throughout this book English units will be used along side SI. Ultimately, it isn’t important which system is used so long as consistency is maintained.

It is important, whenever solving problems that appropriate units are used and recorded. F = 8 is meaningless without units attached. F=8 lb. would be more appropriate. In addition to being descriptive, units can
be used as a check as you perform calculations. First off, you can only add or subtract quantities of the same units. 5 lbs + 8 feet is meaningless. Units can be combined together through the use of multiplication and division. For example, if you multiply 5 lbs and 10 feet

\[ 5 \text{lbs} \times 10 \text{ft} = 50 \text{ ft-lbs} \]

the result will be 50 ft-lbs (pronounced “fifty foot pounds”). This result is now a new unit, no longer a foot or a pound, but a foot-pound. If you were then to divide this result by 2 lbs

\[ \frac{50 \text{ ft-lbs}}{2 \text{ lbs}} \]

the result would be 25 ft. The lbs in the numerator are canceled by the lbs in the denominator leaving only feet. If you are solving for a pressure you should expect to have an answer with units of pounds per square inch (lbs/in\(^2\)). If your answer is in pounds per cubic inch, you know you have made a mistake. A way of making this easier to see is to separate the numerical elements of the equation from the units. See sample problem 1.

**Sample problem 1**

\[ P = fA \]

is known as the direct stress formula (more on this later…), where \( P \) = load (how much something weighs), \( f \) = the allowable unit stress and \( A \) = area. This will be discussed in much greater depth later in the text, however, for now solve for the load \( P \) when \( f = 10 \) pounds per square inch and \( A = 10 \) square inches.

\[ P = fA \]

\[ P = 10 \text{ lbs/in}^2 \times 10 \text{ in}^2 \]

this equation can be rewritten as follows. Notice that in\(^2\) is the same as \( \frac{\text{in}^2}{1} \). Anything divided by 1 remains unchanged.

\[ P = 10 \times 10 \times \left( \frac{\text{lb}}{\text{in}^2} \right) \times \left( \frac{\text{lb}}{\text{in}^2} \right) \]

\[ P = 100 \text{ lb} \]

just as one would expect. The load (weight) is measured in pounds.
1 - 4 Symbols and notations

Math is more easily and effectively carried out using symbols. Frequently used symbols follow in table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>addition</td>
</tr>
<tr>
<td>-</td>
<td>subtraction</td>
</tr>
<tr>
<td>× or .</td>
<td>multiplication</td>
</tr>
<tr>
<td>÷ or /</td>
<td>division</td>
</tr>
<tr>
<td>=</td>
<td>is equal to</td>
</tr>
<tr>
<td>≠</td>
<td>is not equal to</td>
</tr>
<tr>
<td>&gt;</td>
<td>is greater than</td>
</tr>
<tr>
<td>&lt;</td>
<td>is less than</td>
</tr>
<tr>
<td>≥</td>
<td>is greater than or equal to</td>
</tr>
<tr>
<td>≤</td>
<td>is less than or equal to</td>
</tr>
<tr>
<td>Σ</td>
<td>the sum of</td>
</tr>
<tr>
<td>Δ</td>
<td>the change in a quantity</td>
</tr>
</tbody>
</table>

The use of standard notation in the field of structural mechanics is somewhat muddied by the lack of constancy currently used. In this work, I will try to remain consistent with the use of symbols and notations. What follows is a list notation used in this book and the meaning that goes along with it. Note that this will not necessarily be the same notation found in other works. Don’t worry that you might not recognize or understand some or all of these terms. They will all be explained during the course of this text.

A Total area of a surface or cross section
b Width of a beam cross section
c Distance from neutral axis to edge of a beam
d Depth of a beam cross section or overall depth of a truss
D (1) Diameter; (2) deflection; (3) Moment arm
e Elongation
E Modulus of elasticity (Young’s modulus)
f Computed unit stress
F (1) Force; (2) allowable unit stress
h Height
H Horizontal component of a force
I  Moment of inertia
K  Effective length factor for slenderness of a column
M  Moment
p (lowercase)  Unit pressure
P (uppercase)  Concentrated load
r  Radius of gyration of a cross section
R  Radius of a circle
t  Thickness
T  Torsion moment
V  (1) Total shear force; (2) vertical component of a force
w  (1) Width; (2) unit of a uniformly distributed load on a beam
Δ (delta)  Change of
Σ (sigma)  Sum of
θ (theta)  Angle
ϕ (phi)  Angle
π  3.1415

This is by no means a complete list of notation used in structural mechanics. If new notation is used, it will be discussed in the section where it is used.
## Review and Summary

<table>
<thead>
<tr>
<th><strong>Statics</strong></th>
<th>The branch of mechanics which studies bodies held motionless by balanced forces.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dynamics</strong></td>
<td>The study of bodies in motion and forces which are dependent upon time.</td>
</tr>
<tr>
<td><strong>Static Equilibrium</strong></td>
<td>The condition upon which all forces in a system balance out so as not to produce any motion.</td>
</tr>
<tr>
<td><strong>Stress</strong></td>
<td>Internal forces produced by structural members which resist forces supplied by outside influence.</td>
</tr>
<tr>
<td><strong>Strength of Materials</strong></td>
<td>The study of the properties of materials which produce resisting forces (stress)</td>
</tr>
</tbody>
</table>
Chapter 2  Review of Algebra and Trigonometry

It is impossible to study forces and strength of materials without the use of mathematics. This chapter is devoted to reviewing basic laws of algebra and trigonometry. It is not meant to instruct someone with no prior knowledge of these subjects, but, no knowledge of mathematics more advanced than what you learned in high school are required. Even though the expressions and equations contained herein are derived using calculus and linear algebra, knowledge of these subjects are not required to make use of these.

While working through this chapter, you might find yourself asking “what the heck does this have to do with building scenery?” Don’t worry, real world examples will come. An effort has been made to keep the examples very generic in this chapter. The material here is the basis for everything else to follow.

2 - 1 Order of Operations

There are a certain set of rules governing algebraic manipulation known as the order of operations. The following rules are very important for if not followed, very different (and wrong) answers will result. For example;

\[ 4 + 3 \times 5 \]

If you add 4 +3 first and then multiply the sum by 5 you will get 35. However, if you multiply 3 times 5 first and take that result and add it to 4 you will get 19. How can this be? Following the order of operations will prevent this unnecessary error.

**ORDER OF OPERATIONS**

1) If the expression contains grouping symbols, such as parentheses ( ) or brackets [ ] or braces { }, do the operations within the grouping symbols first.

Example: \((4+9)\times(3+8)\). You would first do the operations in the parentheses.

\[ 4+9=13 \]
\[ 3+8=11 \]
\[ (13)\times(11)=143 \]

2) Evaluate any exponents.

Example: \(5^3 \times 3^3\)
First evaluate \(5^3 = 25\)
Second evaulate \(3^3 = 27\)
Last multiply the two resultants \(25 \times 27 = 675\)
3) Do all multiplication and division

Example: $5 \times 3 + 7$
First multiple $5 \times 3 = 15$
Last add $7 = 22$

4) Do all addition and subtraction

5) If the expression contains a division bar, evaluate the entire top and bottom before dividing.

Example: $\frac{5 + 9}{8 - 2}$
First evaluate the expressions above and below the fraction bar
$5 + 9 = 14$
$8 - 2 = 6$
Second divide $\frac{14}{6} = 2.33$

Sample problem 1

Evaluate the expression

$$4 + 5 \cdot 2 + 15 \div 3 - 2$$

First do the expression within the parentheses and braces separately

$$4 + 5 \cdot 2$$
$5 \cdot 2 = 10$
$10 + 4 = 14$

$$15 \div 3 = 5$$

The original expression can now be rewritten as such

$(14) + 5 - 2 = 17$
Sample problem 2

Evaluate the expression

\[
\frac{(15 + 4) \times 3}{(9 \div 3 + 2) \times 2}
\]

First, apply the order of operations to the top portion of the fraction bar.

\[
15 + 4 \times 3
\]
\[
15 + 4 = 19
\]
\[
(19) \times 3 = 57
\]

Second, apply the order of operations to the bottom portion of the fraction bar.

\[
9 \div 3 + 2 \times 2
\]
\[
9 \div 3 = 3
\]
\[
3 + 2 = 5
\]
\[
(5) \times 2 = 10
\]

Lastly, perform the division indicated by the fraction bar.

\[
\frac{57}{10} = 5.7
\]

2 - 2 Rules Governing Equations

An Equation is an expression of equality between two quantities. The quantities on each side of the equal sign, by definition, must be the same (hence the term equal!). In order for the equal sign to remain accurate, if something is changed on one side of the equal sign, the same thing must be done to the other side. Algebraic operations (addition, subtraction, multiplication, division and some others) may be performed to any equation in any way, so long as the exact same operation is performed on both sides of the equal sign. By doing this we change the value of the numbers on each side, but not the fact that both sides of the equation are equal (i.e. the same number). This rule will become very important for us when we begin to solve for unknown quantities in equations. This is a very important idea as it will be the basis for much of the math we will be doing later in this text.
For example;

\[ 5 \cdot 2 = 10 \]

the following operations can be performed without changing the equality of the

\[ 2 \cdot 5 = 10 \cdot 2 \]
\[ 20 = 20 \]

\textit{or}

\[ 5 \cdot 2 + 8 = 10 + 8 \]
\[ 18 = 18 \]

\textit{or}

\[ \frac{5 \cdot 2}{3} = \frac{10}{3} \]
\[ 3\frac{1}{3} = 3 \frac{1}{3} \]

You will notice that the value does change, but the equality of the equation stays true.

There are some basic rules which we can use to solve algebraic equations (Industrial Algebra and Trigonometry, p.31-32). The goal is to manipulate the equation using algebraic operations so as to have the variable by itself on one side of the equation and all of the numbers combined together on the other side.

1. Operations in parentheses, exponents, multiplications, divisions, additions, and subtractions in the usual order of operations.

\[ 5 + 9 \times X = 21 \]
\[ 14X = 21 \]

2. Like terms may be combined. By like terms we mean multiples of the same variable.

\[ 4x + 9x = 53 \]
\[ 13x = 53 \]

3. The same quantity may be added or subtracted to both sides of an equation without changing its value.

\[ X + 2 = 5 \]

first, subtract 2 from each side of the equation
\[ X + 2 - 2 = 5 - 2 \]
\[ X = 3 \]

You probably knew this on an intuitive level. Ask yourself, “what (X) plus 2 equals 5?” The answer, of course, is 3. 3 plus 2 equals 5.

4. Multiplying both sides of an equation by the same quantity does not change its value, so long as you do not multiply by 0. For this example, we are choosing 2. It works with any number, however by choosing 2, the unknown (X) is left alone. More on this later. Notice below that rather than writing \( \frac{1}{2} \times X \) you see \( \frac{1}{2}X \). Whenever you see a number followed by a variable (in this case X) multiplication is implied. \( \frac{1}{2}X \) is a simpler way of writing \( \frac{1}{2} \times X \).

\[
\frac{1}{2}X = 4
\]

\[
2 \left( \frac{1}{2}X \right) = 4 \times (2)
\]

\[ X = 8 \]

Again, you can ask yourself the question “one half of what (X) equals 4?”

5. Dividing both sides of an equation by the same quantity does not change its value, so long as you do not divide by 0. Again, 2 is chosen to divide both sides by.

\[
2X = 4
\]

\[
\frac{2X}{2} = \frac{4}{2}
\]

\[ X = 2 \]

6. Transposition - Any term may be transposed from one side of the equality symbol to the other, provided the algebraic sign is changed, without altering the value of the equation.

\[ X + 5 = 3X \]

is the same as

\[ X = 3X - 5 \]
Note that this is essentially the same as rule 1. In this case, simply subtract 5 from both sides of the equation.

7. Cross Multiplication - If two fractions are equal, the fractions may be eliminated by “cross multiplication,” i.e., equate the product of the numerator of the left member and the denominator of the right member with the product of the numerator of the right member and the denominator of the left member.

\[
\frac{6}{x} = \frac{3}{4} \\
3 \cdot X = 6 \cdot 4 \\
3X = 24
\]

2 - 3 Solving an Equation

By strictly following the rules set forth in 2-2 you will never change the validity of any equations. You can multiply both sides of every equation you encounter by 5 with no fear of making the equation no longer valid. However, with appropriate choices of addition, subtraction, multiplication and division you can start moving toward a solution to the variable. With the rules given in 2 - 2 we can solve most any equation. The goal when trying to solve an equation is to use the above rules to manipulate the equation, without changing its value, so that the variable is by itself on one side of the equality symbol and only a number appears on the other side. This means that an equation with more than one unknown variable cannot be solved for a pure number. You can only solve the equation for one variable in terms of the other or others. In order to solve equations with more than one unknown quantity, or variable, you will need an equal number of related equations. For example, if you have an equation with two variables, you will need two related equations in order to solve them. This is a slightly more complicated, which we will return to later.

Sample problem 2

\[
(5x + 32) - 45 = 27 \quad \text{Solve for } x
\]

\[
(5x + 32) - 45 + (45) = 27 + (45) \quad \text{The first step is to add 45 to both sides of the equation. This}
\]

\[
5x +32 = 72 \quad \text{removes the -45 from the left side without changing the equation.}
\]

\[
5x +32 -(32) = 72 -(32) \quad \text{Next subtract 32 from each side to remove the +32 from the left}
\]

13
\[ 5x = 40 \]
\[ \frac{5x}{5} = \frac{40}{5} \]
\[ x = 8 \]

Finally divide both sides by five to leave \( x \) by its self.

You will notice that each step taken is moving one more number away from the left side and combining it with a number on the right side. Notice that we cannot add the “5” of the 5\( x \) with the 32 until after we multiply the “5” with the “\( x \)” (order of operations). At first glance, it might appear that we are breaking the order of operations by doing addition and subtraction before resolving the 5 multiplied by \( x \). However, 5 multiplied by \( x \) is simply 5\( x \), so the order of operations holds. 5 times \( x \) cannot be simplified any further than 5\( x \).

Sample problem 3

Sample problem 4
\[ \frac{4x + 15}{2y} = 12 \]

You must realize that it is impossible to solve this equation for a pure number because there are two variables (\( x \) and \( y \)). The best you can do is solve for \( x \) in terms of \( y \) or \( y \) in terms of \( x \). Let’s solve for \( x \).

First multiply both sides by \( 2y \) to clear the division

\[ \left( \frac{4x + 15}{2y} \right) \cdot 2y = 12 \cdot 2y \]
\[ 4x + 15 = 24y \]

Then subtract 15 from both sides to begin getting \( x \) by itself

\[ (4x + 15) - 15 = 24y - 15 \]
\[ 4x = 24y - 15 \]

Then divide both sides by 4 to make \( x \) by itself.

\[ x = \frac{24y - 15}{4} \]
\[ x = 6y - 3.75 \]

Remember that order of operations must be followed. You cannot add the 4 from the 4\( x \) and the 15 because you must perform the multiplication (4 times \( x \)) first. Of course 4 times \( x \) simply equals 4\( x \) and that is as far as it can be carried out.

With practice this process becomes second nature. Don’t feel bad if it you are somewhat rusty. Just remember you can do any algebraic operation just so long as you do the operation to both sides of the equation. Not every operation will get you closer to isolating the variable you wish, but it isn’t “wrong” either.
2 – 4 Simultaneous equations

It has been stated that an equation with multiple variables cannot be solved for a pure number. However, this is not entirely true. In order to solve an equation with multiple variables, you need multiple related equations. For example, if you have two equations with two unknowns, they can each be solved, however, if you had two equations with three unknowns, they could not. The trick to solving simultaneous equations is to solve one equation in terms of one variable as in sample problem 3. Then, with one variable separated out, you can substitute the result into the second equation. See sample problem 4.

Sample problem 4.

Given these two related equations, solve for x and y

\[ 4x + 2y = 16 \]
\[ \text{and} \]
\[ 4x + 3y = 18 \]

\[
\begin{align*}
4x + 2y &= 16 \\
(4x + 2y) - 2y &= 16 - 2y \\
x &= 16 - 2y
\end{align*}
\]

\[
\begin{align*}
\frac{4x}{4} &= \frac{16 - 2y}{4} \\
x &= 4 - \frac{1}{2}y
\end{align*}
\]

Note that there are two equations and two unknowns. Therefore each equation can only be solved in terms of the other variable. However, as there are the same number of related equations as unknowns, we can solve for each variable.

First, solve one equation in terms of either x or y

\[
\begin{align*}
4x + 3y &= 18 \\
x &= 4 - \frac{1}{2}y
\end{align*}
\]

\[
\begin{align*}
4 \left(4 - \frac{1}{2}y\right) + 3y &= 18 \\
16 - 2y + 3y &= 18
\end{align*}
\]

\[
\begin{align*}
16 + y &= 18 \\
(16 + y) - 16 &= 18 - 16 \\
y &= 2
\end{align*}
\]

Next, in the second equation, remove any occurrence of x and replace it with what x is equal to.
-2y and 3y can be added together because they are both functions of y. Then, it is possible to solve for y.

Now that we have a value for y, we can substitute this numerical value back into the first equation and solve for x.

2 - 5 Converting between fractions and decimals

A fraction is simply unresolved division. ½ really means, “one divided by two”. We all know intuitively that ½ is 0.5, but what about 15/32? To convert to a decimal, simply divide 15 by 32. To go the other way, when given a decimal to convert to a fraction, first you must decide how accurately you need the fraction to be represented. The smaller the fraction (i.e. the larger the denominator) the more accurate your translation will be. In most cases accuracy to 1/16 inch is all that is necessary for common scenic construction, but the following method works just the same with any fraction. If 1/16 is your accuracy, then multiply the decimal by the denominator, 16 in this case. The result will be how many sixteenths the decimal represents. For example, to convert 0.34 into a fraction, first I decide what accuracy I want and I choose 1/16. I then multiply 0.34 by 16, the denominator of my chosen accuracy. The result is 5.44. We round this down to the nearest whole number (in this case 5) because 5.44 is closer to 5 than to 6. This means that 0.34 is equivalent to 5/16. If I had chosen 1/32 accuracy I would have multiplied 0.34 by 32, which would have given me 10.88. We round this up to 11 because 10.88 is closer to 11 than 10. The result is that 0.34 is equivalent to 11/32 (see sample problem 2). As a check, you can resolve the division. 5/16 = 0.3125. This is close to .034 but not exactly. 11/32=0.34375 which is much closer to 0.34. You can see that 11/32 more closely represents 0.34 than 5/16 and is therefore more accurate. The individual circumstances will help determine how much accuracy will be needed.

Sample problem 4

Convert 0.3856 into a fractional form. Use accuracy to the nearest 1/16.

0.3856 • 16 = 6.1696

or

6/16 \rightarrow 3/8
Sample problem 5

The actual outside diameter measurement of 1 ½" sch 40 pipe is 1.9".
Of course, your tape measure is in fractional inches. What is 1.9" in fractional form? Be accurate to 1/32".

\[ 1.9 \times 32 = 60.8 \]

or

\[ \frac{60.8}{32} = 1 \frac{29}{32} \]

if you do not wish to have to simplify the fraction, you can simply remove the decimal part of the number and perform the same calculations.

\[ 0.9 \times 32 = 28.8 \]

which is rounded to 29

Remember to put the 1 back in!

\[ 1 \frac{29}{32} \]

It is also common to have to convert a decimal number of inches into feet and inches. This is a similar process as is described above. First divide the number of inches by 12 (the number of inches in a foot). This will give you the number of feet and decimal parts of a foot. Take just the decimal part (subtract the whole number) and multiply it by 12 to give you the number of inches and decimal parts of an inch. Next take just the decimal part again (subtract the whole number) and multiply it by the denominator of the accuracy you decide upon. This might seem like many steps, but it is really very simple.

Sample problem 5.

Convert 112.364 inches into feet and inches. Be accurate to 1/16 inch.

\[ \frac{112.364}{12} = 9.3637 \]

9 feet

\[ 0.3637 \times 12 = 4.3644 \]

9 feet 4 inches

\[ 0.3644 \times 16 = 5.8304 \]

9 feet 4 \( \frac{6}{16} \) inches or \( \frac{3}{8} \) inches
2 - 6 The Summation Symbol

There is a mathematical symbol used in mathematics known as the summation symbol. It is shown with the capital Greek letter sigma (Σ). In its basic form, it means to add all of the terms which follow. For example, if \( L \) represents 4, 50 lb. loads then

\[
\sum L = 50 + 50 + 50 + 50 = 200 \text{ lb.}
\]

In addition to this basic use, there are some qualifying notations used in conjunction with the summation symbol which make it more descriptive. A small variable (usually an \( x \)) written below the symbol itself is a starting place holder. It is used when there are a number of values to be summed, but only some are to be used. For example;

\[
\sum_{x=1} L = L_1 + L_2 + L_3...
\]

This means to sum all values of \( L_x \) beginning with \( L_1 \) and continuing to the end. This notation might be used if there are a number of different loads. Perhaps \( L_1 \) is a 50 lb. load, \( L_2 \) is a 70 lb load and \( L_3 \) is a 400 lb load.

\[
\sum_{x=1} L_x = L_1 + L_2 + L_3 = 50 + 70 + 400 = 520 \text{ lbs}
\]

In addition, a number might appear above the summation symbol, indicating the stopping point. For example;

\[
\sum_{x=3}^5 L_x
\]

This means to sum all values of \( L_x \) beginning with \( L_3 \) and ending with \( L_5 \). For example, if \( L_3 = 20 \text{ lb} \), \( L_4 = 50 \text{ lb} \), \( L_5 = 200 \), \( L_4 = 35 \), \( L_5 = 600 \).

\[
\sum_{x=3}^5 L_x = L_3 + L_4 + L_5 = 200 + 35 + 600 = 835 \text{ lb}
\]
2 - 7 Trigonometry

Trigonometry is the mathematical study of triangles. It might seem odd to limit your study to one specific shape, but it turns out to be an incredibly useful shape! Three connected lines form a unique triangle, the sum of the interior angles of which will always total 180°. In other words, if you add up the measurement of the three angles which form a triangle they will always add up to 180°. Triangles play a very important role in the analysis of structures. Before we can attempt to solve any problems involving trigonometry, we first must obtain a common vocabulary.

In a Right Triangles, (triangles with one 90° angle) each side of the triangle has a name. The longest side of the triangle is known as the hypotenuse. The other two sides are denoted as to how they relate to a given angle. They are called side adjacent and side opposite the given angle (see figure 1).

There are three very important ratios between the three sides of a Right Triangle. There is the ratio between the side opposite (O) to the angle and the hypotenuse (H) which is called the sine (abbreviated sin). There is the ratio between the side adjacent (A) to the angle and the hypotenuse which is called the cosine (abbreviated cos). Finally there is the ratio between the side opposite and the side adjacent the angle which is called the tangent (abbreviated tan). They are written mathematically as follows;

\[
\begin{align*}
\sin \theta &= \frac{O}{H} \text{ or Side opposite / Hypotenuse} \quad \text{(Eq.1)} \\
\cos \theta &= \frac{A}{H} \text{ or Side adjacent / Hypotenuse} \quad \text{(Eq.2)} \\
\tan \theta &= \frac{O}{A} \text{ or Side opposite / Side adjacent} \quad \text{(Eq.3)}
\end{align*}
\]

Sine, cosine and tangent are treated as mathematical functions, similar to addition, subtraction, multiplication and division in the sense that they are operations on a number. For example, if you are given an angle of 30°, you can look-up the corresponding value of the sin 30°, cos 30° or tan 30° in a table, or on a scientific calculator. Also, like addition, subtraction, multiplication and division they can be manipulated algebraically. Earlier, when discussing how to solve equations we did the following example;

\[5x + 32 = 72\]

To solve this equation we subtracted 32 from both sides of the equation.
Perhaps you didn’t realize it at the time, but this first step of solving the equation involved performing the *opposite* function of a function in the equation, mathematically known as the *inverse*. Subtraction is the *inverse* of addition. The next step in solving this equation would be to divide both sides of the equation by 5 because 5 is being multiplied by x. Division is the *inverse* of multiplication.

\[
\frac{5x}{5} = \frac{40}{5}
\]

\[x = 8\]

Because **sin**, **cos** and **tan** are mathematical functions, they have *inverses* as well. They are known as **arcsin**, **arccos** and **arctan**. They can also be written as \(\sin^{-1}\), \(\cos^{-1}\) \(\tan^{-1}\). These inverse functions will be vital in solving equations which involve trigonometric functions.

The other unique property of Right Triangles is that the lengths of the sides of the triangle are simply related by the Pythagorean Theorem. The Pythagorean Theorem states that the length of the hypotenuse squared is equal to the sum of the lengths of the sides squared.

\[
A^2 + O^2 = H^2 \quad \text{(Eq.4)}
\]

or

\[
H = \sqrt{A^2 + O^2} \quad \text{(Eq.5)}
\]

Where \(A\) = one side of the triangle

\(O\) = the other side of the triangle

\(H\) = the hypotenuse of the triangle

You might be more familiar with the following form

\[
A^2 + B^2 = C^2
\]

where \(A\) and \(B\) are the legs of the triangle and \(C\) is the hypotenuse of the triangle.

Now that we know equations for sine, cosine, tangent and the Pythagorean Theorem, we can discover many things about a given triangle even if we are only given two pieces of information. If we know the length of two sides we can calculate the third and then all angles that make up the triangle. If we know one angle (and since we are only talking about right triangles we always know one additional angle, the \(90^\circ\) angle) and one side we can calculate the
length of all of the other sides and all of the angles. The thing that must always be remembered is that the definition of sine, cosine, tangent and the Pythagorean Theorem will **only** work with regards to right triangles.

**Sample problem 6**

Given the adjacent Right Triangle, calculate the lengths of all sides and all angles.

From Eq.1

$$\sin q = \frac{O}{H}$$

$$\sin 30 = \frac{O}{12}$$

The value of sin 30 can be determined from your calculator

$$0.5 = \frac{O}{12}$$

$$O = 6$$

From Eq.4

$$12^2 = 6^2 + O^2$$

$$144 = 36 + O^2$$

$$O = 10.4$$

The sum of all angles in a triangle must be 180° (this is the definition of a triangle. See earlier in this chapter) and it is a Right Triangle (one 90° angle).

\[
\hat{A} \quad q + f + 90
\]

\[
30 + f + 90 = 180
\]

\[
f = 60
\]

The final angle must be equal to 60°.

What happens if the triangle in question is not a Right Triangle? We cannot use these definitions of sin, cos, tan or the Pythagorean Theorem as they only apply to Right Triangles, therefore we must find new tools to use. We must also find a new method of labeling sides and angles of triangle for non-right triangles.
First let’s discuss a labeling system so that we all are speaking the same language. Each intersection is labeled with a lowercase letter (a, b, c). The angle will be denoted with the lower case letter which corresponds to the intersection which forms the angle. The sides will be denoted with the capital letter of the intersection directly opposite the intersection. See figure 2.

Now that we have a way of talking about sides and angles of a non-right triangle, we can discuss the tools used when studying non-right triangles.

The Law of Sines says that the relationship between the length of a triangle’s side and the sin of the angle opposite that side is a constant as shown in Eq.6.

\[
\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \quad \text{(Eq.6) Law of Sines}
\]

Here we have three related equations. Three related equations are solvable with up to three unknown variables. Therefore, if we know any two sides and one angle or any two angles and one side, we can calculate all other sides and angles.

The second tool at our disposal is The Law of Cosines. The law of Cosines is the basis for the Pythagorean Theorem. It states that

\[
C^2 = A^2 + B^2 - 2AB \cos c \quad \text{where} \quad A, B, C = \text{legs of triangle} \quad \text{and} \quad c = \text{angle opposite side } C \quad \text{(Eq.7)}
\]

We can see that if the triangle in question were a right triangle then c would be 90°. The cos 90° = 0, hence the law of Cosines reduces to the Pythagorean Theorem. This law is mostly used when you have been given all lengths of sides and need to find the angles.
Sample problem 7.

Solve for all lengths of sides and angles in triangle abc.

First we need to calculate angle c. The sum of all interior angles of a triangle must be 180°. Therefore

\[50° + 70° + c = 180°\]

\[c = 60°\]

From Eq.6 we have

\[
\frac{10}{\sin 70} = \frac{B}{\sin 50} = \frac{C}{\sin 60}
\]

\[
\frac{10}{.940} = \frac{B}{.766} = \frac{C}{.866}
\]

10.638 = \frac{B}{.766} = \frac{C}{.866}

We can separate these three equations into a form that is easier to see

10.638 = \frac{B}{.766}

and

10.638 = \frac{C}{.866}

\[B = 8.15\]

\[C = 9.21\]

Sample problem 8

Find all angles in triangle abc.

From Eq.7 we have

\[
\frac{12}{\sin 7} = \frac{6}{\sin 12}\]

\[
\frac{12}{.198} = \frac{6}{.207}
\]

12.16 = 6.85

\[B = 71.6°\]

\[C = 71.6°\]
\[ C^2 = A^2 + B^2 - 2AB \cos C \]
\[ 12^2 = 7^2 + 6^2 - 2(7)(6)\cos C \]
\[ 144 = 49 + 36 - 84\cos C \]
\[ 144 = 85 - 84\cos C \]

Note that we cannot subtract the 84 from the 85. The order of operations says that we must perform the multiplication \((84 \cos C)\) before subtraction.
\[ 144 - 85 = 85 - 85 - 84\cos C \]

Subtract 85 from both sides of the equation
\[ 59 = -84\cos C \]

Note the negative 84\(\cos C\). Both sides of the equation must be divided by -84
\[ \frac{59}{-84} = -\frac{84}{-84} \cos C \]
- 0.702 = \(\cos C\)

Remember, the inverse of \(\cos\) is \(\cos^{-1}\). By applying \(\cos^{-1}\) to \(\cos C\) you will be left with simply \(c\) (\(\cos^{-1}\) and \(\cos\) cancel each other out).
\[ \cos^{-1}(-0.702) = c \]

You will need your scientific calculator to perform \(\cos^{-1}(-0.702)\)
\[ 134.6^\circ = c \]

Repeat this process for the other two angles.
\[ 7^2 = 12^2 + 6^2 - 2(12)(6)\cos a \]
\[ 49 = 144 + 36 - 144\cos a \]
- 131 = -144\(\cos a\)
\[ 0.9097 = \cos a \]
\[ \cos^{-1}(0.9097) = a \] (remember \(\cos^{-1}\) is the inverse of \(\cos\))
\[ a = 21.5^\circ \]

Remember, the sum of all the interior angles of a triangle must be 180°
\[ a + b + c = 180 \]
\[ 21.5 + b + 134.6 = 180 \]
\[ b = 20.9^\circ \]
Order of operations
1) Grouping symbols; 2) Exponents; 3) Multiplication and division; 4) Addition and subtraction;
Top of division then bottom of division bar then divide.

Rules governing equations
Any operation you perform to one side of the equal sign, you must perform to the other side of the equal sign to preserve unity.

Solving an equation
You must have as many equations as you have unknowns.

Converting decimals to fractions
Multiply the decimal by the denominator of the fractional accuracy you have decided upon.

Trigonometry
Three intersecting lines form a unique triangle. The sum of the interior angles of a triangle total 180°

\[
\sin \theta = \frac{O}{H} \text{ or Side opposite / Hypotenuse}
\]

\[
\cos \theta = \frac{A}{H} \text{ or Side adjacent / Hypotenuse}
\]

\[
\tan \theta = \frac{O}{A} \text{ or Side opposite / Side adjacent}
\]

The Pythagorean Theorem
\[
A^2 + B^2 = C^2 \text{ or } C = \sqrt{A^2 + B^2}
\]

Non-right triangle trigonometry
Law of Sines
\[
\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}
\]

Law of Cosines
\[
C^2 = A^2 + B^2 - 2AB \cos c
\]
Examples and Problems

1. \(2x + 55 = 3x\) solve for \(x\).
2. \((5x + 2) + 12 = 15\) solve for \(x\).
3. \(\sin q = 0.5565\) solve for \(q\).
4. \(x^2 + 20 = 14\) solve for \(x\).
5. \(\frac{7.25x - 14}{15} = 107\) solve for \(x\).

5. Given the right triangle in figure 5 (a), solve for the unknown side. What is the unknown angle? Is the unknown angle necessary to solve for the unknown side?

6. Given the triangle in figure 5 (b), solve for the unknown sides. Is it possible that this is a right triangle?

**Figure 5. Problem 5 and 6**

7. A speaker needs to be rigged 20'-0" below the grid. However its placement requires that it is suspended 12'-0" stage left of a support and 14'-6" stage right of another support. What are the lengths of the cables necessary to rig a bridal to hang the speaker (see figure 6). Kind of looks like a triangle, huh?

**Figure 6. Problem 7.**
Chapter 3   Vectors

3-1 Definition of a vector

A vector is a quantity that has both a magnitude (amount) and a direction. Magnitude is simply a quantity such as 15 feet or 100 pounds or 30 seconds. Some physical quantities that can be represented as vectors are such things as force, velocity or displacement (movement). For example, a force can be described as 15 pounds acting to the right as in Figure 1 or velocity could be described as 30 miles per hour north. Not all quantities involve a direction. Some simply have a magnitude. These are called scalars. Scalars are the type of numbers which you are probably most used to. Examples of scalars are length, mass or temperature, none of which have a direction associated with it. Scalars are manipulated using standard rules of algebra. Vector numbers, however cannot be manipulated as simply. Vectors have their own special algebraic operations. They can be added, subtracted, multiplied by a number or have their direction changed. These are similar to standard operations, but different as we will see.

The simplest of all vectors is the displacement vector. The displacement vector describes movement of a particle (fancy physics term for some object) from one point (or place) to another. In figure 2, there are examples of three displacement vectors, all of which can be treated as if they are identical. For all practical purposes, they are the same. The arrows $\vec{AB}$ (vectors are always denoted in bold face or with a line or arrow drawn above) for vector (1), (2), (3), and (4) all represent the same change in position. The fact that they start in different places and take different routes does not matter. They are the same vector. The vector shows us the result of the motion, not necessarily the motion itself. For another example of displacement, imagine you are traveling from Las Angeles to New York City. The displacement vector would be an arrow drawn from L.A. to NYC even though there are many options as to how you could get there. Perhaps you take a more northerly driving route across the country, stopping to see the Grand Canyon and maybe a side trip to Niagara Falls before arriving in New York. Or Perhaps you take a more southerly route through Phoenix, Dallas and then on to New York. Of course you could take a direct flight from LA to New York. In all of these cases, the displacement vector is the same, even though you might take very different routes.
3.2 Vector Addition - Graphic method

Vectors do not follow the same algebraic rules as do scalars (A.K.A. normal numbers). Suppose we have a car which moves from point A some distance east to point B and then some distance north to point C. The result of its travel is a vector from A to C. We can represent the car’s motion (or displacement) as a vector \( \mathbf{AB} \) followed by a vector \( \mathbf{BC} \), in other words vector \( \mathbf{AB} \) plus vector \( \mathbf{BC} \). The total displacement is the vector \( \mathbf{AC} \). This vector is known as the vector sum or resultant. It represents the addition of vectors \( \mathbf{AB} \) and \( \mathbf{BC} \). This is demonstrated graphically in figure 4.

Vectors can be added in two different ways. They can be added graphically or they can be added using their components. First the graphic method will be discussed. As was said earlier, a vector is made up of both a magnitude and a direction. The magnitude is the amount of vector there is, such as 15 pounds or 7 feet. The direction can be described in any convenient way such as north, up, to the left, etc. For consistency, and ease of mathematics, we should devise a mathematical “map” to use in describing vectors. The most useful map is the standard X-Y plane. \( 0^\circ \) is typically horizontal and to the right along the X axis (see figure 3). \( 90^\circ \) is straight up along the Y axis (see fig. 3). For example vector \( \mathbf{A} \) could be 300 lbs at \( 90^\circ \), which would mean a 300 lb. force pushing or pulling upward. If we have two vectors, \( \mathbf{AB} \) (125 lb. at \( 0^\circ \)) and \( \mathbf{BC} \) (200 lb. at \( 90^\circ \)) we can add the two together to get \( \mathbf{AC} \), the vector sum or resultant. Consult figure 4. To add two vectors graphically, first place the arrowhead of one vector (in this case \( \mathbf{AB} \)) at the tail of the second vector (\( \mathbf{BC} \)). We can do this because, as stated earlier, the start and end points of a vector do not matter, what matters are its magnitude and direction only. We can graphically move vectors anywhere we want so long as we do not change the value of the magnitude (often represented by the length of the arrow) or the angle the arrow is drawn in the X-Y axis. The resultant, \( \mathbf{AC} \), will be the vector that connects the tail of the first vector to the head of the second vector. If each of these vectors is drawn to scale (any convenient scale will due) you could simply measure the length of \( \mathbf{AC} \) in scale. This would be the resultant or sum of \( \mathbf{AB} \) and \( \mathbf{AC} \). Although this is mathematically simpler, it is not always practical. Because the triangle in question is a right triangle, we can use the Pythagorean Theorem to find the hypotenuse, which will be...
the magnitude, just as if the number of pounds were a measurement of length (see figure 5).

\[ \text{AC} = \text{AB} + \text{BC} \]

Notice the bold face text showing that the quantities in question are vectors.

\[ AB^2 + BC^2 = AC^2 \]
\[ 125^2 + 200^2 = C^2 \]
\[ C = 235.8\text{lb} \]

Of course, the magnitude is only half the story. We still need to know what angle AC makes in the X-Y plane.

Next, making use of the definition of tangent to find the angle AC makes with the horizontal vector, AB

\[ \tan A = \text{opposite / adjacent} \]
\[ \tan A = \frac{200}{125} \]
\[ \tan A = 1.6 \]
\[ \text{tan}^{-1}(\tan A) = \text{tan}^{-1}1.6 \]

Remember that \( \text{tan}^{-1} \) is the inverse function of tan, leaving you simply with A.
\[ A = 58^\circ \]

You can add up as many vectors as you wish in this way. Simply continue laying vectors head to tail and using some simple trigonometry.

Often times, we will be dealing with vectors which act parallel with the x-axis or y-axis. As a way of simplifying the vector addition, a positive or negative sign will be applied to the magnitude of the vector to indicate this direction. A vector pointing straight up (90°) or directly to the right (0°) will be considered positive and a vector pointing down (270°) or to the left (180°) would be negative. For example, a horizontal force vector of 10 pounds, pointing to the left (180°) would be notated as negative 10 pounds. This convention is especially useful when adding or subtracting two vectors which do not form a triangle, see figure 6. For example, both AB and CD have magnitudes of 30 pounds; however AB is acting at 0° and therefore has a positive value. CD on the other hand is acting at 180° and therefore has a negative value. Because these two vectors do not form a triangle, we cannot use the methods above but they can still be added.

\[ \text{AB} + \text{CD} = 30 + (-30) = 0 \]

This is an example of static equilibrium. We have a system of two force vectors, each of which have equal magnitudes, but are acting in opposite directions and therefore result in 0. Imagine two people of equally matched playing tug-of-war. One person (represented by vector AB) is pulling equally as hard as the other person.
(represented by vector \( \mathbf{CD} \)). The end result, or resultant will be that nobody goes anywhere. We will see in the next section where this convention comes from and why it works.

It should be noted that equations 1 thru 4 from Chapter 2 – 7 can only be used if the vectors involved meet at right angles. However, very often vectors will meet at angles other than 90°. To graphically add two vectors that do not meet at 90°, we need to make use of the Law of Sines and the Law of Cosines.

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{(Eq. 5 Law of Sines)}
\]

\[
c^2 = a^2 + b^2 - 2ab\cos C \quad \text{(Eq. 6 Law of Cosines)}
\]

Sample problem 1.

What is the sum of vector \( \mathbf{A} \), 1000 lbs at 45° and vector \( \mathbf{B} \), 1500 lbs at 75°? Because vector \( \mathbf{A} \) and \( \mathbf{B} \) do not meet at 90°, we cannot use either the Pythagorean Theorem or the definition of Sine or Cosine. Rather, we will make use of the Law of Sines and the Law of Cosines. First, however, we will redraw our vectors head to tail (see figure 7). The vector \( \mathbf{C} \) is the vector sum, or resultant of vectors \( \mathbf{A} \) and \( \mathbf{B} \). This problem can now be treated as a simple trigonometry problem. The interior angle at B is 150° (by using definition of complementary angles and opposite interior angles) [add this to review of trig in earlier chapter]. Using the Law of Cosines, we can determine the magnitude of \( \mathbf{C} \).

\[
b^2 = a^2 + c^2 - 2ac \cos B
\]

\[
b^2 = 1500^2 + 1000^2 - 2(1500)(1000)\cos B
\]

\[
b = 2418.3 \text{ lbs}
\]

Now that we know the magnitude of \( \mathbf{AC} \) to be 2418.3 lbs, we can use the Law of Sines to determine the remaining angles.
\[
\frac{a}{\sin A} = \frac{b}{\sin B}
\]

\[
1500 = \frac{2418.3}{\sin 150}
\]

\[
\sin A = .31
\]

\[
A = 18^\circ
\]

Notice however, that 18° is the angle vector AC makes with vector AB. We are interested in the angle AC makes with the X axis. By observation we can conclude that the angle AC makes with the X axis is the found by adding the angle between AC and AB (18°) with the angle AB makes with the X axis (45°) for a total of 63°. The complete answer is AC=2418.3 N at 63°.

3 - 3 Vector components

Up to now we have described a vector as a magnitude and a direction given by the angle the vector makes in the x-y plane. Another way of describing a vector is using the vectors’ components. The components of a vector are two vectors, one parallel to the X axis and one parallel to the Y axis which, when added together will result in the starting vector. This may seem like simply an extra step at first. Why convert one vector into two separate vector components, only to add them together again? The big advantage, as we will see, is that adding vectors in component form is often times simpler and quicker than adding them graphically, especially when dealing with more than two vectors. Vector components are commonly noted as \(A_x\) and \(A_y\), where A is the magnitude of the vector and the subscripts x and y refer to the portion of vector A which is parallel to the x or y axis.

Take a close look at vector A in figure 8. For the sake of generality, we will assign vector A with a magnitude of A and a direction of \(\theta\) in the standard X-Y plane. To discover the value of the horizontal and vertical components of A, we can use the definition of sine and cosine.

\[
\cos \theta = \frac{adj}{hyp} \quad \sin \theta = \frac{opp}{hyp}
\]

\[
\cos \theta = \frac{A_x}{A} \quad \sin \theta = \frac{A_y}{A}
\]
with some algebraic manipulation, we can put these into the following more useful form.

\[
\begin{align*}
A_x &= A \cos \theta \\
A_y &= A \sin \theta \\
\end{align*}
\] (Eq. 1, 2)

where \( \theta \) is the angle the vector makes in the X-Y plane. Depending on the value of \( \theta \) the component of the vector may be positive, negative or zero. For example, if the magnitude of \( A \) is 50 pounds and \( \theta \) is 30°,

\[
A_x = 50 \cos 30° \\
A_y = 50(0.866) \\
A_x = 43.3 \text{lbs}
\]

However if the magnitude of \( A \) is 50 pounds and \( \theta \) is 120° (see figure 9).

\[
A_x = 50 \cos 120° \\
A_x = 50(-0.5) \text{ notice the negative sign cos120°} \\
A_x = -25 \text{lbs} \\
A_y = 50 \sin 120° \\
A_y = 50(0.866) \\
A_y = 43.3 \text{lbs}
\]

This is the source of the signing convention discussed in the previous section. A vector pointing up or to the right is positive while a vector pointing down or to the left is negative. Let’s look at an example of a vector pointing directly to the right.

\[
A_x = A \cos q
\]

Because the vector in question is directly to the right, \( q = 0° \)

\[
A_x = A \cos 0° \\
A_x = A(1) \text{ the cosine of } 0° = 1 \\
A_x = A
\]

Just as we would expect. The horizontal component of a vector parallel to the x-axis is simply the magnitude of the vector.
However, if the vector in question is pointing directly to the left

\[ A_x = A \cos \theta \]

Because the vector in question is directly to the left, \( \theta = 180^\circ \)

\[ A_x = A \cos 180^\circ \]
\[ A_x = A(-1) \text{ the cosine of } 180^\circ = -1 \]
\[ A_x = -A \]

The horizontal component of a vector parallel to the x-axis but pointing left is simply the negative magnitude of the vector.

The same exercise works for vectors pointing up or down, with vectors pointing straight up being the positive magnitude of the vector and pointing straight down being negative magnitude of the vector.

\[ A_y = A \sin \theta \]

Because the vector is pointing up, \( \theta = 90^\circ \)

\[ A_y = A \sin 90^\circ \]
\[ A_y = A(1) \]
\[ A_y = A \]

The vertical component of a vector parallel to the y-axis and pointing up is equal to the magnitude of the vector.

And if the vector in question is pointing straight down

\[ A_y = \sin \theta \]

Because the vector is pointing down, \( \theta = 270^\circ \)

\[ A_y = A \sin 270^\circ \]
\[ A_y = A(-1) \]
\[ A_y = -A \]

The vertical component of a vector parallel to the y-axis pointing down is simply the negative magnitude of the vector.

It should also be noted that vector components behave as scalars and the algebraic sign (+ or -) maintains the direction. This means that we can use standard algebraic operations on vector components. Of course, you must make sure to keep the components separate. You can only add x component vectors to other x component vectors and y component vectors to other y component vectors! Once we have converted a vector into its components, we can make use of either notation. Both systems contain the exact same information only communicated in different ways.
Example Problem.

Add the three following vectors using their components.

\[ \mathbf{A} = 50 \text{ lbs at } 45^\circ, \quad \mathbf{B} = 95 \text{ lbs at } 30^\circ, \quad \mathbf{C} = 25 \text{ lbs at } 155^\circ \]

First convert each vector into their respective components

\[
\begin{align*}
A_x &= A \cos q = 50 \cos 45^\circ = 35.35 \text{ lbs} \\
A_y &= A \sin q = 50 \sin 45^\circ = 35.35 \text{ lbs} \\
B_x &= B \cos q = 95 \cos 30^\circ = 82.27 \text{ lbs} \\
B_y &= B \sin q = 95 \sin 30^\circ = 47.5 \text{ lbs} \\
C_x &= C \cos q = 25 \cos 155^\circ = 22.65 \text{ lbs} \\
C_y &= C \sin q = 25 \sin 155^\circ = 10.58 \text{ lbs}
\end{align*}
\]

Now, to add these three vectors, we simply add together each x component and each y component making sure to keep any positive or negative signs.

\[
\begin{align*}
R_x &= A_x + B_x + C_x = 35.35 + 82.27 + (-22.65) = 94.97 \text{ lbs} \\
R_y &= A_y + B_y + C_y = 35.35 + 47.5 + 10.58 = 93.43 \text{ lbs}
\end{align*}
\]

We can easily switch back and forth between vector components and magnitude and direction descriptions. Using a variation of the Pythagorean Theorem and the definition of tangent we note that

\[
A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1} \frac{A_y}{A_x} \quad \text{(Eq. 3 and 4, see figure 10)}.
\]

Figure 10. Combining components back into a vector.
This is essentially the same process we used in section 3-2 with the added benefit that both of the vectors we will be adding (the $x$ and $y$ components of the vector) will always meet at 90°! They must because that is the definition of the $x$ and $y$ component. Using the example problem above we found the resultant in the $x$-plane, $R_x = 94.97\text{lbs}$ and the resultant in the $y$-plane, $R_y = 93.43\text{lbs}$. Using equations 3 and 4, let’s see how to convert these components back into a vector with magnitude and direction.

First to find the magnitude of the vector

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{(94.97)^2 + (93.43)^2}$$

$$R = \sqrt{9019 + 8729}$$

$$R = \sqrt{17748}$$

$$R = 133.2\text{lbs}$$

this is only half the story. We also need to know what direction the force is acting in

$$\tan q = \frac{R_y}{R_x} = \frac{93.43}{94.97} = 0.984$$

$$q = \tan^{-1}(0.984)$$

$$q = 44.54°$$

A special note must be made here. When studied closely, one notices that for any ratio $A_y / A_x$ there are two angles $\theta$. One is the angle that your calculator will give you and the other is $180°$ greater. Try it. The $\tan 44.54°$ is 0.984. The $\tan 224.54°$ ($180° + 44.54°$) is also 0.984. When calculating an angle a vector makes in the $X$ - $Y$ plane, some common sense must be used to determine if it is the angle from the calculator or $180°$ greater. This can most easily be determined by drawing the two components of the resultant to get an idea as to which direction it is going. In the case of our example, we see that both $R_x$ and $R_y$ both have positive values. A positive value in the $x$ direction indicates a vector pointing to the right and a positive value in the $y$ direction indicates a vector point up. Even without knowing the values of these two vectors, we can conclude that the resultant of these will be a vector pointing up (positive $y$ value) and to the right (positive $x$ value). $44.54°$ points both up and to the right, where $224.54°$ points down and to the left. See figure 11.

![Figure 11](image-url)
3 - 4 Vector addition - Component method

The graphical method is useful to an extent but cumbersome, especially in three dimensions or with many vectors to add. Now that we have the ability to resolve vector components we can develop a more straightforward and practical method. The rule for vector addition is simple: (1) resolve the vectors into their components; (2) add the components for each like axis; and (3) if necessary, recombine the components into a vector. Simple algebraic addition is all that is necessary, as the vector components work like scalars. You can usually simplify this process by choosing your coordinate system wisely. For example, most of what we will cover can be studied in two axis, therefore eliminating the need for any Z component. If the x-axis is going left and right, the y-axis going up and down, then the z-axis is going into and out of the page. This might be somewhat difficult to visualize and can usually be avoided in all but the most complicated structural analyses.

Example problem 2.

A car drives 36 miles east of the gas station. It then drives 45 miles north. Finally it drives 25 miles northwest (135° or 45° from the horizontal). Figure 12 shows the route. At the car’s destination, how far along a straight line from the gas station is it?

This is simply a vector addition problem. You could choose to use the graphical method to solve it, but you will find that it is easier to break the given vectors into components and use the component method.

First write each vector in terms of its components.

\[ a_x = a \cos \theta \]
\[ a_y = 36 \cos 0 \]
\[ a_z = 36 \text{ miles} \]
\[ a_x = a \sin \theta \]
\[ a_y = 36 \sin 0 \]
\[ a_z = 0 \text{ miles} \]

It should be intuitively obvious that \( a_z = 0 \) as the vector a acts entirely in the X axis. You should also see that \( b_z = 0 \) as the vector b acts entirely in the Y axis.
\[ b_x = b \sin \theta \]
\[ b_y = 45 \sin 90 \]
\[ b_y = 45 \text{miles} \]
\[ c_x = c \cos \theta \]
\[ c_x = 25 \cos 135 \]
\[ c_x = -17.7 \text{miles} \]
\[ c_y = c \sin \theta \]
\[ c_y = 25 \sin 135 \]
\[ c_y = 17.7 \text{miles} \]

Note that 45° could be substituted for 135° in relation to vector \( c \). If you do however, you need to realize that \( c_x \) acts in the negative X direction and is therefore a negative number.

Now that we have the three vectors broken down into their components, we can use simple algebraic addition to add all the \( x \) components and \( y \) components to get the components of the resultant vector.

\[ r_x = a_x + b_x + c_x \]
\[ r_x = 36 + 0 + (-17.7) \]
\[ r_x = 18.3 \text{miles} \]

\[ r_y = a_y + b_y + c_y \]
\[ r_y = 0 + 45 + 17.7 \]
\[ r_y = 62.7 \text{miles} \]

after making use of equations 3 and 4

\[
r = \sqrt{r_x^2 + r_y^2} = \sqrt{(18.3)^2 + (62.7)^2} = 65.3 \text{miles}
\]

and \( \theta \) can be found by:

\[
\theta = \tan^{-1} \frac{r_y}{r_x}
\]

\[
\theta = \tan^{-1} \frac{62.7}{18.3}
\]

\[
\theta = \tan^{-1} 3.43
\]

\[
\theta = 74^\circ
\]

or 65.3 miles at 74° from the start point.
**Review and Summary**

**Definition of vectors and scalars**
Scalars, such as temperature or mass have magnitude only. They obey normal rules of algebra. Vectors, such as displacement or force, have both a magnitude and direction (ex. 5 miles, north) and obey special rules of vector algebra.

**Vector addition - Graphic**
Two or more vectors may be added graphically by laying them head to tail and using rules of trigonometry to solve for their angles and magnitudes.

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{Law of Sines}
\]

\[
c^2 = a^2 + b^2 - 2ab \cos C \quad \text{Law of Cosines}
\]

**Vector components**
\[a_x = a \cos \theta \quad a_y = a \sin \theta\]

Given components, we can reconstruct the vector from

\[a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad q = \tan^{-1} \left( \frac{a_y}{a_x} \right).\]

**Vector addition - Component**
Sum the components for each like axis. Resultant components can be recombined to a vector with magnitude and direction.
Exercises and problems

3 - 2 Vector addition, graphic method

1) Graphically find the resultant of the following vectors. All angles given in relation to a 360° format.

   a) \( \text{a} = 125 \text{ lb. at } 15°, \text{b} = 200 \text{ lb. at } 45° \)
   b) \( \text{a} = 450 \text{ lb. at } 135°, \text{b} = 300 \text{ lb. at } 180° \)
   c) \( \text{a} = 35 \text{ ft. at } 50°, \text{b} = 15 \text{ ft. at } 0°, \text{c} = 40 \text{ ft. at } 35° \)

3 - 3 Vector components

2) Resolve the following vectors into their components.

   a) \( \text{a} = 200 \text{ lb. at } 15°, \text{b} = 300 \text{ lb. at } 45° \)
   b) \( \text{a} = 345 \text{ lb. at } 135°, \text{b} = 543 \text{ lb. at } 180° \)

3) Given the following vectors components, combine to form a vector with a magnitude and direction

   a) \( a_x = 150 \text{ lbs} \quad a_y = 225 \text{ lbs} \)
   b) \( a_x = 15 \text{ miles} \quad a_y = 23 \text{ miles} \)
   c) \( a_x = 34.5 \text{ lbs} \quad a_y = 22 \text{ lbs} \)

3 - 4 Vector addition, component method

4) Using the vector component method, solve for the resultant of the following vectors.

   a) \( \text{a} = 235 \text{ lb. at } 15°, \text{b} = 185 \text{ lb. at } 45°, \text{c} = 95\text{lbs at } 90° \)
   b) \( \text{a} = 437 \text{ lb. at } 135°, \text{b} = 224 \text{ lb. at } 180° \)
   c) \( \text{a} = 75 \text{ lb. at } 315°, \text{b} = 100 \text{ lb. at } 70°, \text{c} = 35 \text{ lbs at } 140° \)
Chapter 4  Forces

The structures we build must perform the task of withstanding forces applied to them. These forces can be the result of many things; most often in our field the forces involved are due to gravity. A platform must hold up the weight of the actors standing upon it. A flat must hold up the weight of itself. On rare occasions, our structures must withstand other forces. For example, a theatrical style flat must be able to withstand the force of the sized muslin pulling toward the center of the flat as the muslin shrinks. Other forces we are concerned with are the stresses applied by structural members of structures. For example, a standard 4 x 8 platform (framed in 2x4, covered in ¾” plywood…) with 6 legs might need to hold up 4 performers. Let’s say that each performer weights 200 lbs for a total of 800 lbs. Assuming that all 4 performers aren’t standing on top of each other, each leg will be asked to hold up 133.33 lbs. plus the weight of the platform itself. Analyzing the forces involved in a system is critical in determining if a piece of scenery will stand up or not. This will always be the first step in determining what materials will be necessary to withstand these forces. Unfortunately, the real world rarely presents itself as a simple word problem. This analysis will be discussed in more detail later in this chapter. Remember, a force is a vector and therefore has both a magnitude and a direction and is subject to vector algebra.

4 - 1  Static Equilibrium

For most of our work we are studying systems which are in equilibrium, that is to say, systems where all of the forces (due to gravity and the forces created by the structural members supporting the loads) are balanced. When forces aren’t balanced, the object that the force is acting on will move due to the unbalanced force. For example, if you push on a platform sitting on the floor, the force of friction between the platform and the floor will push against you. If you are not strong enough to counter the frictional forces (i.e. the platform is too “heavy”), the force of you and the force of friction will be balanced and the platform will not move. If you are strong enough to overcome the force of friction, the platform will slide across the floor. Typically load bearing scenery (such a platform) does not want to be moving on its own, under the influence of gravity (i.e. falling). This situation is generally known as failure (ach!). When structural members of scenery are not capable of resisting the forces being applied to it, it fails, usually falls down and destroys the illusion that what the audience is looking at is real (or at best looks foolish).

In order to have a system in equilibrium the net force acting on it must be zero. In other words, when all of the forces involved are added together, they will add up to zero. In order for any sum of numbers to add to zero, some of those numbers must be negative. Figure 1a and b show such a

![Figure 1. Static Equilibrium](image-url)
system. In figure 1a, \( F_1 \) and \( F_2 \) are equal in magnitude and opposite in direction. Because the system is in static equilibrium, mathematically we can say that;

\[
\sum F = F_1 + F_2 = 0
\]

Remember the summation symbol (\( \sum \))?

In order for this to be true \( F_1 \) and \( F_2 \) must be equal in magnitude and one must be a negative number. We see that if we resolve these two vectors into their components we will obtain:

\[
F_{1x} = F_1 \cos 0^\circ = F_1 (1) = F_1
\]

and

\[
F_{2x} = F_2 \cos 180^\circ = F_2 (-1) = -F_2
\]

and because

\[
\hat{\mathbf{a}} F = F_1 + F_2 = 0
\]

\[
F_1 = -F_2
\]

This is a very simplified case as both forces are acting parallel to the X-axis. We can perform the same investigation in the up / down (Y) direction.

\[
F_{1y} = F_1 \sin 0^\circ = F_1 (0) = 0
\]

and

\[
F_{2y} = F_2 \sin 180^\circ = F_2 (0) = 0
\]

This shows us that the y component of both forces \( F_1 \) and \( F_2 \) are zero. You could probably see this intuitively as neither \( F_1 \) nor \( F_2 \) are pushing up or down in any way.

Remember, A horizontal force to the right is said to be positive while a horizontal force to the left is said to be negative. A vertical force up is said to be positive and a vertical force down is said to be negative.

In Figure 1b we see that there is a total of 300 lbs. (150 lbs +150 lbs.) pushing up and an equal 300 lbs pushing down. Remembering that y axis vectors pointing up are positive and y axis vectors pointing down are negative we have;

\[
150 + 150 + (-300) = 0
\]

Since the sum of the forces in both examples in Figure 1 are 0 the systems we can say that the system is in static equilibrium, or just equilibrium.
Formally, static equilibrium is written as follows:

\[
\begin{align*}
\sum F_v &= 0 \\
\sum F_h &= 0 \quad \text{Eq. 1} \\
\sum M &= 0
\end{align*}
\]

which states that the sum of the forces in the vertical direction and the sum of the forces in the horizontal direction must both sum to 0 for there to be an equilibrium state. The last statement \( \sum M=0 \) refers to the sum of the moments must be 0. More on this later.

Sample problem 1.

Three confused stagehands are pushing on a wagon. Dave (A) is pushing with a force of 100 lbs at an angle of 25°. Jason (B) is pushing with a force of 25 lbs at an angle of 45° and Mary (C) is pushing with a force of 185 lbs. at 250°. Is this system in equilibrium? If not, what vector D is necessary to make it so?

What this question is really asking is “Does the sum of these three forces equal 0? If not, what would it take to make them sum to zero?” To find out, we can add these three together graphically, or break them into components. As there are more than two forces involved, components should prove to be the easier solution.

Using Eq. 1 and 2 from Chapter 3

\[
\begin{align*}
A_x &= 100 \cos 25 = 90.6 \text{lbs} \\
B_x &= 25 \cos 45 = 17.7 \text{lbs} \\
C_x &= 185 \cos 250 = -63.3 \text{lbs} \\
A_y &= 100 \sin 25 = 42.3 \text{lbs} \\
B_y &= 25 \sin 45 = 17.7 \text{lbs} \\
C_y &= 185 \sin 250 = 173.8 \text{lbs}
\end{align*}
\]

Using Eq. 1 from Chapter 4
The resultant of these three forces 45 lbs to the right (0°) and 113.8 lbs down (270°). As this result isn’t 0, we can safely say that this system of forces is not in equilibrium. This wagon is going to be moving as a result of the stagehands pushing. In order to put it in equilibrium we need a fourth force D, to be 45 lbs to the left (180°) and 113.8 lbs up (90°). Converting these components back into a vector;

\[ R = \sqrt{F_x^2 + F_y^2} \]  
(from Eq. 3 Chapter 3)

\[ R = \sqrt{45^2 + 113.8^2} \]

\[ R = \sqrt{2025 + 12950.44} = \sqrt{14975.44} \]
\[ R = 122.37 \text{ lbs} \]

\[ q = \tan^{-1} \frac{F_y}{F_x} \]  
(from Eq. 3 Chapter 3)

\[ q = \tan^{-1} \frac{113.8}{45} \]
\[ q = \tan^{-1}(-2.53) \]
\[ q = -68.4° \]

Negative 68.4°? What is a negative angle? Is our wagon entering hyperspace or something? Remember, our angle measurements are based on the Cartesian coordinate system where 0° is to the right, 90° is up, 180° is to the left and 270° is down. Negative 68.4° simply means measuring counter-clockwise rather than clockwise. See figure 2.

4 – 2 Moments

A moment is defined as the tendency of a force to cause rotation about a point or an axis. It is also sometimes referred to as torque, however this is somewhat of a misnomer. Both torque and moment refer to the tendency of a force to produce rotation, however torque is usually used in relation to something that is actually turning.
such as the wheel of a car or a wrench on a bolt head. A moment, in contrast, is purely the tendency of the force to produce rotation. For example, imagine you are standing on a board overhanging the edge of the table. If someone holds down the other end of the board you might not fall on the floor. The moment is a measurement of how hard the person will have to hold down the other end. The board will still have a tendency to rotate (and dump you on the floor) even if it doesn’t actually move. A torque is a type of moment, however you can have a moment without necessarily having torque. See figure 3.

The magnitude of the moment is given by the magnitude of the force multiplied by the perpendicular distance to the point the rotation is centered upon.

\[ M = Fd \]

where \( M \) = Moment
\( F \) = Force
\( d \) = perpendicular distance from pivot point to location where the force is acting (a.k.a. moment arm)

The most common unit of measurement for moments is the foot-pound (pronounced “foot pound”), however this is entirely based on what units you use to calculate the moment. In figure 3 we use 1 foot and 15 pounds, therefore we have a moment of 15 ft-lbs. If the units were 15 tons and 1 yard we would have 15 yard-tons (ok, you will probably never see yard-tons!). The point here is that the units for moments are simply the combination of the units used in calculating the moment.

Sample Problem 2:

You have a 4’x8’ platform overhanging 3 feet from the edge of another platform. A 200 lb actor stands right on the edge of this overhanging (or cantilevered) platform. What is the moment acting on the platform?

In this case, the moment arm is 3 feet. The 200 lb actor is standing 3 feet from the edge where the platform will tend to rotate around. The force is the force of gravity pulling the 200 lb actor down.

Using equation 2

\[ M = Fd \]

\[ M = 200lb(3\ ft) \]

\[ M = 600\ ft-lb \]
Remember, to satisfy static equilibrium we must have the sum of the moments be zero in addition to the sum of the vertical and horizontal forces.

\[ \sum M = 0 \quad \text{Eq. 3} \]

Let us also state that a moment tending to cause clockwise rotation is said to be negative and a moment tending to cause counterclockwise rotation is said to be positive. This convention is entirely arbitrary. So long as your sign convention is consistent, the math will work out correctly.

**Sample Problem 3**

Given the situation from Sample Problem 2, what is the minimum force is needed (how heavy a person) to prevent the platform from tipping over if the force is applied to the opposite end of the platform? Disregard the weight of the platform itself.

The real question here is what force is needed to maintain static equilibrium? When both forces are balanced the platform will not rotate.

From equation 3 we have
\[ \sum M = 0 \]
which is necessary to maintain equilibrium. We have 2 moments working.

\[ M_1 = \text{the moment due to the actor standing over the edge} \]
\[ M_2 = \text{the moment necessary to prevent the actor from falling} \]

\[ \sum M = M_1 + M_2 = 0 \]
\[ 600(\text{ft} - \text{lbs}) + M_2 = 0 \]
\[ M_2 = -600(\text{ft} - \text{lbs}) \]

From equation 2
\[ M = Fd \]
\[ F = \frac{M}{d} = \frac{600(\text{ft} - \text{lbs})}{5(\text{ft})} = 120\text{lbs} \]

Notice that the negative sign is dropped. When calculating the force produced by a moment, the absolute value is used. The negative sign in the moment is simply saying that the moment will produce a tendency to rotate in a clockwise direction.
4 – 3 Force Systems

When it comes to studying force systems, it is often advantageous to draw the systems being studied so as to better visualize what is going on. There are some standard symbols used which should be discussed.
The Uniformly Distributed load represents the case where the load is spread out over the length of a beam in an even way. Imagine stage weights laid end to end along a horizontal 2x6 for example. The distributed load is given by the letter “w” and will be in units of weight per unit length, such as lb/ft. If the stage weights all weigh 40 lbs and are 1 foot long, we would say that the 2x6 has a uniformly distributed load of 40 lb/ft.

The Single Point Load is used when you have a load concentrated in a single localized area. In the real world, nothing occupies a true “point” in the mathematical sense, so we make some approximations. For example a 150 pound actor standing on a platform would be treated as a point load of 150 lb, even though his or her weight is spread out over the area of both of their feet.

The next three symbols, Pinned, Roller and Fixed connections deal with how a beam or other support is connected to other support systems. A pinned or hinged connection is capable of doing two things. It will hold the end of the beam up and it will prevent the beam from moving left and right. Imagine a piece of steel tubing bolted to a steel plate flange (see figure??). Unless the bolt breaks or one of the two steel pieces break the bolt will prevent the steel tubing from moving up / down and left / right. Of course the bolt doesn’t really prevent the steel tubing from rotating (never mind trying to make the bolt really tight). In this case we say that the pinned connection provides Reactions in both the x and y planes, but does not resist a moment that might be applied to the steel. This is shown to the right in the diagram above by the vectors $R_H$ and $R_Y$ which represent the horizontal reaction and vertical reaction. For the roller connection, imagine a wagon corner being supported by a caster. The caster will hold up the wagon, but the wagon is still capable of moving left and right. In the case of a roller connection, we say that the connection provides a vertical reaction only, represented by the vector $R_Y$. The final connection, the fixed connection is one that is not often seen when building scenery. You can imagine the fixed connection being a steel or wooden beam that has been imbedded in a concrete wall when the concrete was poured. The fixed connection provides reactions to both horizontal and vertical forces ($R_H$ and $R_Y$) as well as resistance to moments, M.

Many force systems we are likely to encounter can be reduced down to forces acting in two planes by the appropriate choice of coordinate systems. Forces acting in two planes are known as coplanar. Non-coplanar forces get much more complicated and will not be covered within this text.

Coplanar forces can be broken into three different classes;
1. concurrent forces – multiple forces acting on the same point
2. non-concurrent forces – multiple forces acting on different points of the same mass
3. parallel forces – multiple forces acting parallel to each other, but not acting on the same point.

Figure 4 shows an example of each.
**Resolving two concurrent forces**

Resolving (adding or subtracting) two concurrent forces can be solved by either the graphic method, or through the method of components. Often, the graphic method is simpler, especially if the two forces meet at right angles. The examples provided in chapter 3 were all examples of resolving concurrent forces, even though this term was not used.

Sample Problem 2

Two stagehands are pulling ropes tied to a wagon with four sets of swivel casters. Due to a lack of communication the two stagehands are not pulling in the same direction. Stagehand A is pulling the wagon directly stage left (let’s call this 0°) with a force of 50 lbs and Stagehand B is pulling slightly upstage (let’s say 25°) with a force of 75 lbs. See figure 5. Are these two forces concurrent? What is the resultant vector?

The solution can be arrived at by using either the graphic method or the method of components. Using the graphic method, first we need to arrange the two vectors head to tail. See Figure 6.
The angle of $155^\circ$ is arrived from subtracting $25^\circ$ (the angle that \( \mathbf{B} \) makes with the x-axis) from $180^\circ$.

Using the Law of Cosines

\[ c^2 = a^2 + b^2 - 2ab \cos C \]

where \( a \), \( b \) and \( c \) are the sides of the triangle and \( C \) is the angle opposite side \( c \). In our example \( R \) is side \( c \)

\[ R^2 = 50^2 + 75^2 - 2(50)(75)(\cos 155) \]
\[ = 2500 + 5625 - 7500(-0.906) \]
\[ = 8125 - (-6795) \text{ notice that subtracting a negative number is the same as addition } \]
\[ = 8125 + 6795 \]
\[ R^2 = 14920 \]
\[ R = 122.15 \text{ lbs} \]
Of course, we could also use the method of components.

First, convert both vectors into their components

\[ a_x = a \cos \theta \]
\[ a_y = a \sin \theta \]

\[ A_x = 50 \cos 0 \]
\[ A_x = 50 \text{lbs} \]
\[ A_y = 50 \sin 0 \]
\[ A_y = 0 \text{lbs} \]

\[ B_x = 75 \cos 25 \]
\[ B_x = 67.97 \text{lbs} \]
\[ B_y = 75 \sin 25 \]
\[ B_y = 31.70 \text{lbs} \]

\[ R_x = A_x + B_x \]
\[ R_x = 50(\text{lbs}) + 67.97(\text{lbs}) \]
\[ R_x = 117.97 \text{lbs} \]

\[ R_y = A_y + B_y \]
\[ R_y = 0(\text{lbs}) + 31.70(\text{lbs}) \]
\[ R_y = 31.70 \text{lbs} \]

After converting the resultant components back into vector form

\[ R = \sqrt{R_x^2 + R_y^2} \]
\[ R = \sqrt{(117.97(\text{lbs}))^2 + (31.70(\text{lbs}))^2} = \sqrt{13916.9 + 1004.89} \]
\[ R = \sqrt{14921.79} \]
\[ R = 122.15 \text{lbs} \]

Just as we would expect.

**Resolving three or more concurrent forces**

When there are three or more concurrent forces, it is usually simpler and more straightforward to apply the method of components to sum the forces. It is possible to solve it graphically; however the trigonometry tends to be more involved. The process is exactly the same as it is for resolving two concurrent forces using the component method. First resolve all of the given vectors into their components, add their respective components and then finally convert the component sums back into a single vector.
Resolving parallel force systems

When there are multiple parallel forces acting on a body and these forces do not pass through a common point it is known as a parallel force system. The magnitude and direction of the resultant can be determined by the summation of the forces. Remember, the only thing that is important to the vector is the magnitude and direction. Assuming that the object the forces are acting on is ridged and unbending (we will always make this assumption) the location in which they act on an object is irrelevant! By making this assumption we can simply add all of the parallel forces together using the signing convention discussed previously.

Sample Problem;

Imagine a 270 pound person standing at the center of an 8’ long beam. The beam is supported by legs on either end. Assuming the beam does not bend (okay, it will bend but not so much that it matters for now. Later we will look at how to determine if the beam bends too much….) it stands to reason that each of the legs is holding up half the weight of the person, or 135 pounds.

\[ w = 270 \text{ lbs.} \]

\[ r = 135 \text{ lbs.} \]

\[
\sum F_x = 0 \quad \text{there are no forces acting in the x direction}
\]

\[
\sum F_y = w + r + r = 0
\]

\[
w = -(r + r) = -2r
\]

\[
-270(lbs) = -2r \quad \text{remember, forces acting down have a negative sign}
\]

\[
135lbs = r
\]

It stands to reason that if the person move to the left, the left support will be taking more of the load. Taken to the extreme, if the person is standing directly above the left support, the left support will be carrying the entire load and the right support will carry nothing. Because we cannot always rely on loads to be centered or directly above supports, we need a mathematical method to determine the load on each support.
In parallel force systems, it is sometimes useful to replace multiple given point loads with a single vector that represents an equivalent force and location. The magnitude and direction of the resultant force can be determined by summing the given forces. The position of this resultant force may be determined by making use of the principle of moments. This position will be given by the sum of the moments of the given forces.

For example, consider three loads on a horizontal beam as shown in figure 5.

**Sample problem.**

Determine the resultant of the parallel forces acting on the beam AB in Figure 5. Ignore the weight of the beam.

The resultant R will be the vector sum of the three vectors shown. Using Eq. 1 and 2 from chapter 3 you can discover the value of the vertical component of each of these forces. Or you can realize that because all of the forces are only acting in the vertical direction that the vertical components of each vector will be equal to the magnitude of the vector. You have a 25 pound force pulling up therefore the vertical component of that vector is simply +25 (remember the signing convention?). For review, see chapter 3.

\[
R = \sum F_v = 25 - 30 - 45 = -50
\]

The resultant will be parallel to the original forces (in this case vertical) and the negative sign shows us that it is pointing down. The only question that remains is where along the beam does this reaction act? To uncover this we will make use of the definition of moments (Eq. 2). A moment can be calculated for any force from any point, however there are usually more logical choices for a center of moments point. In this case the left end of the beam, where all of the measurements are made from makes sense.

The first thing to do is determine the value of the moment due to each of the given forces. Remember, a moment is defined as the product of the magnitude of a force and the perpendicular distance from the pivot point. In
this case we chose the pivot point to be the hinged connection on the left (it can be anywhere, we just happened to choose this point. You should see soon why this makes the math easier).

\[ M_1 = 25 \text{lbs} \times 0 \text{ft} = 0(\text{ft} - \text{lbs}) \]

\[ M_2 = 30 \text{lbs} \times 3 \text{ft} = -90(\text{ft} - \text{lbs}) \]

\[ M_3 = 45 \text{lbs} \times 9 \text{ft} = -405(\text{ft} - \text{lbs}) \]

\[ \sum M = -90(\text{ft} - \text{lbs}) - 405(\text{ft} - \text{lbs}) = -495(\text{ft} - \text{lbs}) \]

Notice that \( M_1 \) has no moment as the force passes through the moment center point and has a 0 distance. \( M_2 \) and \( M_3 \) are both negative because they both will cause a clockwise rotation about the left endpoint of the beam and by convention a clockwise rotation will cause a negative moment.

By definition, the sum of the moments of the individual forces must be the moment of the resultant. Therefore

\[ M_R = -50(\text{lbs}) \times x(\text{ft}) = \sum M = -495(\text{ft} - \text{lbs}) \]

\[ -50x(\text{ft} - \text{lbs}) = -495(\text{ft} - \text{lbs}) \]

\[ x = 9.9(\text{ft}) \]

### 4 - 4 Forces and stresses

There are three basic things that can happen to a structural member when a force is applied to it. It can be stretched longer and this is known as a \textit{tensile force} or that the member is in \textit{tension}. The member can be crushed smaller which is known as a \textit{compressive force} or that the member is in \textit{compression}. The third possibility is that the material is forced to slide past adjacent layers of the material. This is known as \textit{shear}. Figure 6 shows an example of each. In each case shown, the molecules in the material (the post, rod or rivet) develop a force to resist the load applied to it. This resisting force is known as the \textit{stress} and is defined as the internal force per unit area of the cross section of the resisting material. In the situation shown, if the compressive post is an 8 x 8 post (true size) then the cross sectional area is equal to 8 x 8 or 64 in\(^2\). There is a total force applied of 6400 lb., therefore the post must supply \( 6400 \text{lbs} / 64\text{in}^2 = 100 \text{ lbs} / \text{in}^2 \). The process is the same for the tensile member and the member in shear.

The general formula for simple direct stress is

\[
 s = \frac{P}{A} \quad \text{or} \quad P = sA \quad \text{or} \quad A = \frac{P}{s} \quad \text{(Eq.4)}
\]

where \( s = \) unit stress for the material in question

\( P = \) applied load

\( A = \) cross-sectional area of the material in question
The direct stress formula will sometimes be written in the following form

\[
f = \frac{P}{A} \quad \text{or} \quad P = fA \quad \text{or} \quad A = \frac{P}{f} \quad \text{(Eq. 5)}
\]

where \( f \) = allowable unit stress for the material in question
\( P \) = applied load
\( A \) = cross-sectional area of the material in question

Another situation for shear is where a load is placed upon a beam, which is supported by a wall on each end. Failure in shear in this case results in the ends of the beam sliding past the portion of the beam supported by the wall as shown in figure 7.

Figure 6. Shear force acting on a beam.

Figure 7. Shear force acting on a beam.
Sample problem 3.

A steel bar has a 38,000 lb. load hanging vertically on it (which makes it a tensile force). If the allowable unit tensile stress for steel is 22,000 PSI, what is the required cross sectional area of the steel rod? What diameter rod would be appropriate?

We have 38,000 lbs. which is our load, or P. The allowable unit stress is 22,000 PSI which is our f. We are asked to solve for area A.

From Eq. 5 we have

$$A = \frac{38,000lbs}{22,000\text{psi}} = 1.73\text{in}^2$$

and the area of a circle is represented by

$$A = \pi r^2$$

$$1.73(\text{in}^2) = 3.1415(r^2)$$

$$r = .74(\text{in})$$

Therefore a length of 1 ½” inch steel rod would be able to handle the required load of 38,000 lbs. Notice that the length of the rod is not a factor in calculating the necessary diameter of the rod. We will see later that in compressive members, the length is very critical.

Sample problem 4.

What is the weight limit of a short (this ONLY works for very short posts) square post, measuring 3 ½ in., by 3 ½ in, if the allowable unit compressive stress is 1000 lbs/in²?

From Eq.1, we know

$$P = fA$$

$$= (1000(\text{lbs/in}^2))(3.5(3.5)(\text{in}^2))$$

$$P = 12250\text{lbs.}$$

Note that this only applies to very short compressive members. For longer members, the length of the member becomes important. This will be discussed later on.

Sample problem 5.

Two lengths of steel strap are bolted together using a single 3/8” bolt. These two lengths of steel strap are being pulled apart with a force of 15,000 lbs. Assuming the shear strength of the bolt is 58,000 psi (pounds per square inch or lbs/in²), will the bolt fail?

The first thing to realize in the problem is that the bolt is being subjected to a shear force as the two steel straps try to slide apart. Now you have two options. You could calculate how much force it takes
to shear a 3/8\" bolt or you could calculate the stress the bolt is being asked to supply and compare that number to the bolt’s strength. First, lets look at how much force it takes to shear this bolt.

The given shear strength of the bolt is 58,000psi, therfore,

\[ f = \frac{P}{A} \]

\[ 58000(\frac{lbs}{in}) = \frac{P}{A} \]

\[ A = \pi r^2 = 3.1415(0.1875)(in^2) \]
\[ A = 0.110C \]

\[ 58000(\frac{lbs}{in}) = \frac{P}{0.110(in^2)} \]

\[ 6380lbs = P \]

This tells us that it takes 6380lbs of force to shear a 3/8\" bolt. The force being applied is 15,000 lbs! I would say the poor bolt stands little chance of surviving this.

The second option works out like this

\[ s = \frac{P}{A} \]

\[ s = \frac{15000(lbs)}{0.110(in^2)} \]

\[ s = 136,363\frac{lbs}{in} \]

The bolt can supply 58,000 psi which is substantially less than the 136,363psi that is being applied to it. Good bye bolt!

4 - 5 Static loads vs. Dynamic loads

Loads on a structure can be classified into two basic types. Static loads and dynamic loads. A static load is one in which a load is applied gently and acts there, unmoving for an extended period of time. For example, a curtain hanging from a batten is a static load. It is simply the weight of the fabric hanging from a batten. A static load can also be an estimation of the weight of the materials used in the construction of the scenic unit. The legs of a platform have to be able to hold up the platform, any flats on top of it as well as the actors on it. Heavy use of plaster texture, medium density fiber board, thick walled steel all are becoming more and more common and their weight should be added into the calculations. The magnitude of the static loads are easily calculated once you know what materials are being used. The problem is to have some idea of the sizes before the analysis has begun. Common sense plays an important role here. You can usually take an educated guess as to the size and type of material before analyzing the structure or you can perform an analysis of the structure neglecting the load due to material. From these results, material load can be estimated and then the structure can be reanalyzed using these new quantities.
Dynamic loads, on the other hand, can be much more difficult to predict. Most of what we do concern ourselves with are dynamic loads although we treat them as static loads for simplicity sake. A dynamic load is one in which the magnitude of the force changes with time. An actor walking down a staircase is a dynamic load. A wagon rolling across stage is a dynamic load. It is important to recognize a dynamic load because typically a dynamic load will impart a greater force to the structure than simply its own weight. Imagine you are standing on a 2 x 4 x 8. Simply standing in the middle of the length of it will probably not cause it to break. However, imagine jumping onto it from a ladder. The 2 x 4 probably wouldn’t be able to stand up to such abuse. Actors moving and dancers dancing all create dynamic loads. Table 1 shows some examples of dynamic loads taken from the architectural industry. Many times we can use these to approximate the use of a scenic unit.

Table 1  Typical dynamic load resistance

<table>
<thead>
<tr>
<th>Occupancy</th>
<th>Load (in pounds per ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apartment</td>
<td>40</td>
</tr>
<tr>
<td>Balconies, fixed seats</td>
<td>50</td>
</tr>
<tr>
<td>Dance halls</td>
<td>100</td>
</tr>
<tr>
<td>Dwellings</td>
<td>40</td>
</tr>
<tr>
<td>Offices</td>
<td>50</td>
</tr>
<tr>
<td>Stairways</td>
<td>100</td>
</tr>
<tr>
<td>Stores, wholesale</td>
<td>75 - 100</td>
</tr>
<tr>
<td>Theatrical stages</td>
<td>125 - 150</td>
</tr>
</tbody>
</table>

(Structural Analysis, P.6)

Notice how high stage floors must be rated. This stems from the fact that stages are notorious for being unpredictable. In the days of vaudeville it was not uncommon for an elephant to be brought on stage and today I wouldn’t be shocked to see a car driven onstage.

Impact load is a sub-category of live loads (along with snow loads, wind loads, earthquake loads…) and is one in which the mathematics involved can get quite cumbersome. The simplest case of an impact load is dropping a block on a beam. The block still has the same weight, but in addition to its weight, it has the additional force of inertia (Structural Analysis, p.11). Inertia is covered by Newton’s second law of motion and is a function of mass and acceleration. Acceleration is a vector and as such has a magnitude and direction. Usually acceleration is thought of as getting faster, but, because of its vector nature, you can have an acceleration vector pointing in the opposite direction of the velocity (also a vector). This would be reducing the velocity or ‘deceleration’. Therefore an actor jumping from one platform down to another would experience a negative acceleration as they landed. Actually being able to calculate this acceleration and hence the force due to inertia is possible with some assumptions taken into account, however the usefulness of such a calculation must be considered. The practical upshot of this is that a 200 lb. actor, jumping down six feet will produce an inertial force of about two tons! (Structural Calculations for the
Stage) This is most likely to be the worst case scenario as an actor will probably not willingly jump down more than six feet onto a hard platform.

4 - 6 Deformations

When a material produces stress to counter some external force, that material deforms to some degree. That is to say it changes shape. It may be crushed shorter, stretched longer or any other variation thereof. In fact it is the deformation that actually causes the stress. As a compressive force is applied to the material, the molecules which make up the material start to be pushed together. The forces of repulsion between the molecules push the molecules apart to where they want to live. This is an oversimplified view of what happens but it is essentially correct. The same physics applies to a material in tension. As the load tries to pull the material apart, the material sets up internal forces to counteract the external forces pulling on it.

Robert Hooke, a mathematician and physicist working in the seventeenth century developed a theory which states that “deformations are directly proportional to stresses.” This is known as Hooke’s law. In other words, if you double the stress on a material, the deformation will double. When the stress is removed, the material will (for the most part) return to its original size and shape. However this theory holds valid only up to a certain point after which it breaks down.

Suppose we have a steel rod with a cross sectional area of 1 in² and put it in a tension measuring machine and measure its length very accurately. After applying 5000 lbs. of force (which corresponds to 5000 PSI in the steel rod, \( s = \frac{P}{A} \)) we note that the steel rod has elongated by some distance which we will call \( x \). Suppose now we double the force to 10,000 lbs. We now note the steel rod has elongated by 2\( x \). So far Hooke’s law holds true. However eventually we will reach a stress where the steel rod will stretch more than \( x \) amount per 5000 lbs. of load. This limit is called the elastic limit or yield stress. For common A36 steel this limit is about 36,000 PSI. The yield stress will be different with different grades of steel and with different materials. Beyond the yield stress the material will not return to its original size and shape and Hooke’s law no longer applies.

After we surpass the yield stress, the material will continue to resist the force applied to it. However, when a sufficient force is applied failure occurs. The stress just before this happens is called the ultimate strength. For A36 steel, the ultimate strength is about 70,000 PSI.

Even though there is a considerable amount of cushion between the yield stress and the ultimate strength, we never design a structure up to the ultimate strength. We always want the structure, under normal conditions, to return to its original size and shape after removal of the load, so therefore we must stay under the elastic limit. In addition to staying within the elastic limit of the material, there must be a safety factor built into our calculations. The minimum safety factor for load bearing scenery should be 2/3 of the yield stress (Simplified Engineering for Architects and Builders). For example, if you are using typical mild steel with a yield stress of 36,000 lbs. you should only use 24,000 lbs. in equations involving it. There is never harm in going a little lower especially if you are not 100% sure of the loads involved.
How much will a material deform? This is a good question and although you usually do not need this information in the design of structural members, it is good to have some idea of how much change in shape a structural member undergoes. The \textit{modulus of Elasticity} is the constant relating the amount of deformation in a member with the load on that member. It is given by the expression

\[
e = \frac{Pl}{AE} \quad \text{(Simplified Mechanics and Strength of Materials) \ (Eq.5)}
\]

where 
- \( e \) = total elongation in inches
- \( P \) = force in pounds
- \( l \) = starting length in inches of the material in question
- \( A \) = cross sectional area in square inches of the material in question
- \( E \) = modulus of elasticity in pounds per square inch of the material in question

For wood, \( E \) ranges from 1,000,000 PSI to 1,900,000 PSI and mild steel is 29,000,000 PSI, so for most reasonable loads, the deformation is not that great. It is important to remember that this equation only holds true when the load is less than the yield stress.

\textbf{Sample problem 5}

A 1 in. diameter steel rod has a 15,000 lb. load applied to it. The rod is 10 feet long. How much will the rod elongate under this force?

First we must determine if this force is within the elastic limit for the steel rod. We will use the equation

\[
s = \frac{P}{A}
\]

where \( P = 15,000 \) lbs. and \( A = .785 \text{ in}^2 \) \( (A = \pi r^2) \) and we find that \( s = 19,108.3 \text{ lbs/in}^2 \). which is well under the acceptable yield stress of steel of 24,000 PSI. Therefore we can use Eq.2 to solve for the amount of elongation.

\[
e = \frac{P}{AE}
\]

\[
= \frac{(15,000 \text{ lbs}) \cdot (120 \text{ in})}{(.785 \text{ in}^2) \cdot (29,000,000 \text{PSI/ in}^2)}
\]

\[
= 0.08 \text{ in.} \quad \text{or} \quad 5/64 \text{ in}
\]

The rod will actually stretch 5/64" longer when the 15,000 lb load is applied to it. Because the stress is less than the yield stress of the steel, when the load is removed, the steel will return to it's original size.
4 - 7 Free body diagrams and force polygons

When attempting to analyze a system, you must first have a standard way of symbolically representing the system on paper. A loading diagram, is essentially a drawing of the structure being analyzed. It shows all structural members as well as all external loads applied to the structural members. As a general rule it will not show details of the structure such as bolts, nails, pulleys etc., nor will it depict the thickness of structural members in any kind of scale. It should be drawn in scale regarding length, i.e.: a member which is ten feet long should be drawn twice the length of a member which is five feet long. However, since you do not know the size of the material in a cross sectional manner, members should not have any kind of thickness.

In a free body diagram, one removes all details from the problem and leaves only symbolic representations of all force vectors involved. These force vectors are derived from the loads affecting the system in addition to the stresses being induced in the structural members of the system. In a free body diagram, you will see what forces are acting in what directions and how they relate to other forces in the system. Figure 8a shows an example of a loading diagram, and figure 8b shows the free body diagram derived from it.

Notice in figure 5 (b) how the elements from the loading diagram are converted into vectors in the free body diagram. Vector $\mathbf{R}$ is the reaction caused by the vector $\mathbf{W}$ and is drawn with equal magnitude and opposite direction to vector $\mathbf{W}$. The reaction represents what the sum of the forces due to the structural members must be in order to resist the load. It is conventionally drawn as a dashed line.

Vectors $\mathbf{AB}$ and $\mathbf{BC}$ represent forces developed by stress in member $\mathbf{AB}$ and $\mathbf{BC}$. For the time being, we will make the assumption that they are two-force members, meaning that the member is subject to equal, opposite and collinear forces. This assumption allows us to say that the forces involved will always act along the length of the member and will always produce either tension or compression, never bending. Look closely at the loading diagram in Figure 5(a). It is assumed that point A is securely fastened to the wall (later we will be able to prove this to
ourselves, but for now lets take it for granted). We must also assume that the wall has sufficient strength so that point C will not be forced through the wall. With these two assumptions, we can now analyze the forces at work in this structure. The load W is going to be pulling straight down on point B by gravity. If point B were free to move the distance between A and B would increase as B fell down. This tells us that the member AB is in tension. The load W is trying to stretch AB apart. To balance this tensional force, AB needs to be pulling on point B, hence the direction of the arrow shown for vector AB in Figure 5(b). Now, look closely at member BC. If point B were free to move, the distance between B and C would decrease. The load W is trying to crush BC, there, BC must push against point B, hence the direction of the arrow shown for vector BC in Figure 5(b). It is important to remember that for our study of Statics the structure must be in equilibrium (ΣF=0) and this can sometimes help you determine if a member is in compression or tension. This is generally a fairly intuitive task, however if you do choose incorrectly you will find, after performing your calculations that a member has a “negative” force in it. All this means is that you choose the incorrect sense (compression or tension) but the magnitude will still be correct.

From the free body diagram, we can generate a force polygon. A force polygon is used to analytically discover the values of the forces in a system. The force polygon for the system in figure 8 is shown in figure 9.

Making use of the properties of vectors develops this force triangle. To create it you disassemble the free body diagram vectors and reassemble them into a polygon form, in this case (and in most cases) the polygon is a triangle. (We will develop additional methods to deal with a system that has many vectors in its force polygon.)

Figure 9. Force polygon (triangle) of system shown in figure 8.
Sample problem 5.

A speaker cluster is going to be hung in an arena setting from roof trusses. The roof trusses are 20'-0" apart and the speaker cluster needs to hang 5'-0" to the left of one roof truss and 20'-0" below the roof trusses.

The speaker cluster will weigh 10,000 lbs. when completely loaded. How much tension will be in the two cables used to hang it?

First we must determine what angles the cables are in relation to the vertical line of the speakers.

From the loading diagram (figure 10) we can draw two separate triangles so as to calculate the angles $\theta$ and $\phi$. See figure 8.
To find the angles $\theta$ and $\phi$, given these two sides, we use tangent.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{20 \text{ feet}}{15 \text{ feet}} = 1.33$$

$\theta = 53^\circ$

$\theta_1 = 37^\circ$

$$\tan \phi = \frac{20 \text{ feet}}{5 \text{ feet}} = 4.00$$

$\phi = 76^\circ$

$\phi_2 = 14^\circ$

Notice that $\theta_2$ and $\phi_2$ are two parts whose sum is the entire angle we need, therefore

$37^\circ + 14^\circ = 51^\circ$

Next we need to draw the free body diagram for this system. We choose the point where all of the forces are congruent. See figure 9.
From the free body diagram, we see that there is a 10,000 lb. reaction to the 10,000 lb. weight which is supplied by the two cables. The tensional force in the two cables are represented by vectors $\mathbf{ab}$ and $\mathbf{bc}$. The free body diagram also allows us to draw the force triangle for this system as shown in figure 10.

![Figure 10. Force triangle.](image)

Notice this is not drawn to scale.

From figure 10, we can use the law of sines to solve for the magnitude of vectors $\mathbf{ab}$ and $\mathbf{bc}$.

\[
\frac{10000}{\sin 129^\circ} = \frac{\mathbf{ab}}{\sin 14^\circ} = \frac{\mathbf{bc}}{\sin 37^\circ}
\]

\[
12867.6 = \frac{\mathbf{ab}}{\sin 14^\circ} = \frac{\mathbf{bc}}{\sin 37^\circ}
\]

$\mathbf{ab} = 3113 lbs.$ \quad $\mathbf{bc} = 7744 lbs.$

Now we know both the magnitude and direction of the two force vectors. They are

$\mathbf{ab} = 3113$ lbs. at $127^\circ$ (or $37^\circ$ to a vertical line)

and

$\mathbf{bc} = 7744$ lbs. at $76^\circ$ (or $14^\circ$ to a vertical line)
Review and Summary

Static Equilibrium

A condition in which the sum of all forces acting within a system produce no motion.

\[ \sum F_v = 0 \]
\[ \sum F_n = 0 \]
\[ \sum M = 0 \]

Direct Stress Formula

\[ f = \frac{P}{A} \]
\[ P = fA \]
\[ A = \frac{P}{f} \]

Static Loads

A load gently applied and acts there, unmoving, for an extended period of time i.e. A curtain on a batten

Dynamic Loads

1) A load of uncertain nature. i.e. How many actors will really be on that platform?
2) Impact load. The additional force a system must withstand to bring a moving object to rest.

Elastic Limit

The stress limit, at which a material will not return to its original size and shape.

or Yield Stress

Most importantly, the stress used in most calculations involving strength of materials. 36,000 lbs / sq. in. for A36 steel.

Ultimate Strength

The stress at which the material can no longer hold itself together.

70,000 lbs / sq. in. for A36 steel.

Elongation

\[ e = \frac{Pl}{AE} \]

The total elongation of a material under a given stress.

\[ e = \text{elongation} \quad P = \text{force in pounds} \quad l = \text{length in inches} \quad A = \text{cross sectional area in square inches} \quad E = \text{modulus of elasticity in pounds per square inch.} \]

Loading Diagram

Symbolic representation of the system being studied

Free Body Diagram

Symbolic representation of forces involved in a system.

Force Polygon

Force vectors from the free body diagram, arranged so as to solve for the unknown forces graphically.
Exercises and Problems

4 - 1 Static equilibrium
1. There are two stagehands, each pushing on the same box, but in opposite directions. The first stagehand (A) is pushing with a force of 50 lb. If the box is not moving, what force must the second stagehand (B) be pushing with. How do you calculate it formally?

4 - 2 Forces and stresses
2. A piece of scenery is hung from a piece of round solid steel. The scenery weighs 550 lb. That is the required area of the steel? What is the required radius of the steel? Assume the value of f to be 22,000 lb/in².
3. Four very short legs support a load of 2000 lb. evenly. What is the quantity of the force that leg must support? How big will a piece of Douglas fir need to be to resist crushing (assume f to be 1150 lb/in²)?

4 - 4 Deformations
4. How much will a piece of 1 in solid square steel bar (modulus of elasticity = 29,000,000 lb/in²) stretch under a load of 20,000 lb.?

4 - 5 Free body diagrams and force polygons
5. Given the structure in figure 11, calculate the forces active in AB and BC assuming AB to be 10 ft. long, BC to be 12 ft. long and W to be 750 lb.
6. Given the structure in figure 11, calculate the forces active in AB and BC assuming AB to be 8 ft. long, BC to be 14 ft. long and W to be 900 lb.

![Figure 11. Problem 5 and 6](image)

7. A 200 lb. person is being supported from two cables. One cable is 30 ft long, the other is 20 ft. long and they are 25 ft apart at the top. Draw the loading diagram for this situation. What is the force in each cable? What happens to the value of the force in the cables as the distance they are apart begins to increase? What happens as the distance begins to decrease?
Chapter 5  Designing Structural Members

We now have a basic understanding of how to determine the magnitude and direction of forces in simple systems. We can take it to the next logical step of how to determine what size and what material should be used to make a structure a stable one.

In this section we will discuss how to best choose between the vast arrays of possible materials available. Common materials include steel and lumber; however it is becoming more and more common to see aluminum alloys due to their high strength and low weight. There are also other more specialized materials such as plastics that we will not be discussing in any great depth; however the basic procedures covered in this text are still used even with these more exotic materials.

5 - 1 Tensional Structural Members

Structural members who are in tension are certainly the simplest type of member to design. We have already designed tensional members in chapter 4 except that you did not know you were doing it.

The only concern when designing a member in tension is the load on it and the cross sectional area of the given material. We use the direct stress formula (Eq.1, chapter 4)

\[
A = \frac{P}{f_T} \quad \text{(Eq.1)}
\]

where \( f_T = 0.60 f_y \)

\( A \) = the required cross sectional area in \( \text{in}^2 \)

\( P \) = load in lbs.

\( f_T \) = allowable unit stress in tension for the given material in \( \text{lb/}_{\text{in}}^2\)

\( f_T \) is a value which can be found in tables in Appendix A and C at the end of this text. We use this form of the direct stress formula as opposed to the other two because most commonly we know how much load we need to support and once we decide what type of material (steel, wood, aluminum…) we are using then we can look up a value for \( f_T \) from a table. It is a question of how big of a piece of the given material will be needed to withstand the load given.

It is not beyond the realm of possibility that you know the size and type of material and you need to find out how much weight can be loaded onto the structure. In this case we will use one of the other forms of the direct stress formula, obtainable with a little algebra (or see Eq.1, chapter 4).

For the sake of simplicity, we will assume, for the time being, that the ends of the member in question are fixed solidly to both the load and whatever is holding the actual member up. Remember that this is only for the sake of learning and in reality the load must be attached to the member in some way and that attachment must be as strong as or stronger than the material holding the load. Also, the member must be attached to something to prevent it from
being pulled along by the force of the load and that connection, also, must be as strong as or stronger than the structural member in question. This is why you will rarely see lumber acting as a tensional member. The connections are difficult to securely fasten the load to the member and the rest of the structure. That is not to say that you will never see this as it does occur in wooden trusses, which we will discuss later.

When designing a member that is determined to be in tension, the first thing to do is to determine the magnitude of the force acting on that member. This can be done using the methods laid out in chapter 4, by using the definition of equilibrium (which will be discussed later) or by any other means. This is the load used in the direct stress formula as shown in sample problem 1.

Sample problem 1.

There is a 5,000 lb. box hanging from the end of a piece of square solid mild steel ($f_Y = 36,000$ lbs/in$^2$). Assuming the ends are securely fixed, how large will the square bar have to be to safely hold the load?

To solve this problem, you simple plug in the 5,000 lbs. for the load ($P$) and use 36,000 lbs/in$^2$ for $f_Y$ to solve for the necessary area.

$$f_T = 0.60 f_Y$$

$$f_T = 21600 \text{ lbs.}$$

$$A = \frac{P}{f_T} \quad \text{(Eq. 1)}$$

$$A = \frac{5000 \text{ lbs.}}{21600 \text{ lbs./in}^2}$$

$$A = 0.227 \text{ in}^2$$

The area of a rectangular section is determined by multiplying its length by its width, which, with a square, is its length squared.

$$A = l^2$$

$$0.227 \text{ in}^2 = l^2$$

$$l = 0.476 \text{ in} \quad \text{or} \quad 15/32 \text{ in.}$$

A piece of 15/32 inch square steel bar. 15/32” bar is somewhat rare to say the least, so why not round it up to 1/2 inch.

There are charts located in appendix B that reveal the cross sectional areas of most common steel and aluminum shapes. It is not necessary to try to calculate the cross sectional areas of these.
Compressive Structural Members (Columns)

Compression and tension are the two most common stresses you are likely to encounter. Any time you put legs under a platform, you have a compressive member. When a structural member is placed in compression, there are two different ways it can fail. First it can simply be crushed. The fibers or molecules that make up the material cannot resist the external force applied to them, so they must find somewhere to be pushed to. Imagine pushing down on a tower of clay. The clay simply squishes out the sides. The second way a compressive member can fail is through buckling. The external force applied to the member causes it to bend in the middle or buckle. Imagine pushing down on the end of a piece of spaghetti. The spaghetti will not crush the way the clay did, but it will still snap in the middle. The first case usually occurs only in very short pieces, where failure due to buckling is more common and is usually found in long lengths.

Failure due to crushing is a simple matter to solve for, but when to use this method is a less simple matter. In most of the work done in theatre this type of failure will not come into play. The problem can be solved again using the direct stress formula.

\[ A = \frac{P}{f} \]

f in this case is equal to 0.66 times the materials yield stress.

More often than not, the length as compared to the cross sectional area will be far too great to simply use the direct stress formula. To design a structural member to resist buckling you must consider a number of things about the system. You must consider the unsupported length of the column, the load applied to the column and the end conditions of the column.

Unsupported length refers to the overall length of the column between bracing. If a column doesn’t have any cross bracing, the unsupported length is the whole length of the column. If there is cross bracing present, then the unsupported length is the longest distance between one end and the bracing. You must remember that this bracing must act in both planes of the column. For example, bracing a leg on a platform parallel to the 4 foot side and not the 8 foot side will only reduce the effective column length in the direction of the cross bracing.

End conditions refer to how the ends of the column are connected to other parts of the system. The ends can be fixed in translation, fixed in rotation, both or neither (see Figure 1). An end being fixed in translation means that that end of the member cannot move laterally in any way, it cannot move left or right. For example a 2x4 leg bolted to a platform frame using one bolt. An end being fixed in rotation simply means that the end cannot rotate about any point. With only one bolt attaching to the leg, the leg cannot move sideways (it is fixed in translation) but it can still spin around that one bolt like an axle. Adding a second bolt prevents it from being able to spin (fixed in rotation).
Resistance to buckling is a function of the placement of material in the cross section. The placement of material is distilled down to a single number known as the radius of gyration. The radius of gyration of an area is difficult to describe and of questionable significance for our purposes anyway. It is expressed by:

\[ r = \sqrt{\frac{I}{A}} \]

where \( r \) = the radius of gyration with respect to a given axis (in)

\( I \) = the moment of Inertia with respect to the same given axis (in\(^4\))

\( A \) = the cross-sectional area (in\(^2\))

There are tables that list the radius of gyration (among other quantities) for most common structural shapes of steel and aluminum.

Appendix A, Chart C gives some simple shapes and the formula for the radius of gyration that is useful for lumber. It must be noted that the radius of gyration is dependent only upon the geometry of the cross section. A piece of solid 2” square steel bar will have the same radius of gyration as a piece of 2” solid balsa wood.

To calculate the required cross section area we begin with the direct stress formula with a slight modification (Eq. 1a). This modification takes into account the column’s ability to resist being crushed as well as it’s ability to resist buckling.

<table>
<thead>
<tr>
<th>Buckled shape of column is shown by dashed lines</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Buckled shapes" /></td>
</tr>
<tr>
<td>Recommended K value</td>
</tr>
<tr>
<td>End condition code</td>
</tr>
</tbody>
</table>

**Figure 1. K values for various end conditions**
Simplified Engineering for Architects and Builders

![Figure 1](image)
\[
A = \frac{P}{f_c} \quad \text{(Eq. 1a)}
\]
where \( A \) = the cross-sectional area (in\(^2\))
\( P \) = the load (lbs.)
\( f_c \) = the allowable unit stress in compression for the chosen material (lbs/in\(^2\))


\( f_c \) must now be calculated, or read off of a chart and is dependent upon the ratio of the unsupported length and the radius of gyration as shown in Eq.2. This ratio is known as the \textit{slenderness ratio}.

\[
\frac{l}{r_{\text{min}}} = \text{slenderness ratio} \quad \text{(Eq. 2)}
\]
where \( l \) = the unsupported length (in.)
\( r_{\text{min}} \) = the minimum radius of gyration

Notice that the slenderness ratio is a quantity without units. Because most wooden members have a rectangular cross section, it is useful to replace \( r_{\text{min}} \) with the rectangular equivalent. Eq.3 shows the slenderness ratio using the minimum width of the side of a solid rectangular stock.

\[
\frac{l\sqrt{12}}{d_{\text{min}}} = \text{slenderness ratio} \quad \text{(Eq. 3)}
\]
where \( d_{\text{min}} \) = minimum thickness of the rectangular member

To design a steel column to resist buckling and crushing, we need to satisfy the modified direct stress formula

\[
A = \frac{P}{f_c}
\]

where \( f_c \) is found in appendix B, table 1 and table 2 and is a function of the slenderness ratio, and the slenderness ratio formula. For steel structures, the slenderness ratio must fall between 1 and 200. 200 has been determined to be the maximum limit of the slenderness ratio for steel by the American Institute of Steel Construction (AISC). A slenderness ratio higher than 200 is considered to reflect a column so slender as to be overly sensitive to uncontrollable factors (Applied Statics and Strength of Materials). However any value for slenderness ratio that falls between 1 and 200 will resist buckling equally well. Therefore we now have Eq.4 that states

\[
200 \geq \frac{l}{r_{\text{min}}} \quad \text{or} \quad 200 \geq \frac{l\sqrt{12}}{d_{\text{min}}} \quad \text{(Eq. 4)}
\]
To design a steel compressive column, both Eq.1a and Eq.4 (later we will see that Eq.4 must be modified slightly) must be satisfied. At the start Eq.1a has two unknowns (f_c and A) one of which depends upon the results of Eq.4 (f_c). Values for f_c in steel can be found in appendix B table 1. The value for the slenderness ratio can be chosen to be any value between 1 and 200. We will see later that some choices are more efficient and cost effective than others, although any choice between those two values will work. With the value of the slenderness ratio chosen, you can solve Eq.4 to find the minimum radius of gyration or minimum thickness. Also with the slenderness ratio chosen, you can look up in appendix A, table 1 or table 2 the adjusted value for f_c, the allowable unit stress in compression. With that information you can solve Eq.1 to find the minimum cross sectional area necessary to withstand the load. The radius of gyration and the cross sectional area for most common structural shapes appears in a chart in appendix B. Any piece of steel that has a greater radius of gyration as well as a greater cross sectional area than those values calculated will support the load. As you study Eq.1 and Eq.4, you should discover the two are interrelated. If you decrease the value of the slenderness ratio, f_c increases. Also, as you decrease the value of the slenderness ratio, r_{min} must increase. By trying different values for the slenderness ratio, you will find a point where the minimum radius of gyration and the required area become more closely matched. Being able to see this takes experience but the following rule of thumb is sometimes helpful:

1. High load and low length will usually lead to a low slenderness ratio.
2. Low load and long length will usually lead to a high slenderness ratio.
3. High load and long length will usually lead to a medium slenderness ratio.

This rule of thumb is useful in helping you decide where to start but you should also investigate a few other possibilities.

So far we have taken into account the length of the column and the load applied to the column. The final thing that must be considered is the end conditions of the column. The end conditions can have a strong effect on how the column supports the load. I think for most it is intuitive that the more secure the column is (fixed in translation AND rotation) the more load the column can hold up. These end conditions become involved in the form of a constant, K, which is multiplied with the slenderness ratio. The modified slenderness ratio equation is shown in Eq.5.

\[
\frac{Kl}{r_{min}} = \text{slenderness ratio} \quad \text{or} \quad \frac{Kl\sqrt{12}}{d_{min}} = \text{Slenderness ratio} \quad (\text{Eq.5})
\]

Simplified Truss Design

K is assigned a unitless number depending upon the end conditions (see Figure 1).
Sample problem 2

A platform with four, 2'-0" legs, has a uniformly distributed load of 3,000 lbs. (the load is evenly divided between all four legs). The top of the legs are fixed in rotation and free to translate and the bottom of the legs are free to rotate and fixed in translation. (This is probably the most common scenario for most platforms. Yes, the leg itself cannot move relative to the platform, but the top of the platform can shift side to side. It is also useful as this leads to a K value of 2.0 which will yield the worst case scenario.)

What size steel square tube will be needed to support this load?

First we need to decide upon a slenderness ratio. Since 750 lbs. (3,000 lbs. divided among 4 legs) is a relatively low load and 24 inches is a somewhat long length, let us start with a guess of 200 for the slenderness ratio. From Eq.5 we have

\[
200 = \frac{Kl}{r_{\text{min}}} \\
200 = \frac{(2.0)(24)}{r_{\text{min}}} \\
r_{\text{min}} = 0.24\text{in}
\]

With the decision of the value for slenderness ratio, we can consult table 1 to find the value for \(f_c\). Eq.1 gives us

\[
f_c = \frac{P}{A}
\]

\[
3730\% = \frac{750\text{lbs}}{A}
\]

\[
A = 0.201\text{in}^2
\]

After consulting the table for square tube in appendix B table 4, we find that ¾ in., 14 gauge box tube has a cross sectional area of 0.2214 in. and a radius of gyration of 0.2746 in.

For a wooden compressive member the process is essentially the same with one exception. We must develop a method to determine the allowable unit stress \(f_c\) since it is impractical to have a chart for every species of wood in every grade. To calculate \(f_c\) we have Eq.6.
\[ f = \frac{0.3E}{\left(\frac{KL}{d_{\text{min}}^2}\right)} \quad (\text{Eq.6}) \]

where:
- \( f_c \) = the allowable stress in compression (\( \frac{\text{lbs}}{\text{in}^2} \))
- \( E \) = the modulus of elasticity (\( \frac{\text{lbs}}{\text{in}^2} \))
- \( K \) = the end condition constant (no units)
- \( d_{\text{min}} \) = the minimum width of a solid member (in)

Timber Construction Manual, p.4-4

In the case for wooden compressive members is still necessary for the slenderness ratio to be contained by the following

\[ \frac{KL}{d_{\text{min}}^2} \leq 50 \quad (\text{Eq.7}) \]

Simplified Truss Design p.336

and the same rules of thumb apply as in the case of steel columns.

Sample problem 3

We have the same platform from sample problem 2, 1000 lb. supported equally by 4 legs, except this time the legs are Douglas Fir lumber. Calculate what size lumber will be necessary to support the load.

First we need to decide upon a slenderness ratio. Let us begin with 30

From Eq.7 we have

\[ 30 = \frac{(2)(24\text{in})}{d_{\text{min}}} \quad d_{\text{min}} = 16\text{in}. \]

With the decision of 30 for the slenderness ratio, we can calculate \( f_c \) from Eq.6

\[ f_c = \frac{(0.3)(1760000\text{lbs/in}^2)}{30^2} \quad f_c = 587\text{lbs/in}^2 \]

and with \( f_c \) used in Eq.1a

\[ A = \frac{750\text{lbs}}{587\text{lbs/in}^2} \quad A = 1.28\text{in}^2 \]

Since we know the dimension of one side (the shortest side) and the area, we can calculate the dimension of the other side.

\[ A = (d_{\text{min}})(d) \]
\[ 1.28\text{in}^2 = (16\text{in})(d) \quad d = 0.80\text{in}. \]
This tells us that we need a piece of Douglas fir, 1.6 inches by 1.6 inches because \( d \) cannot be less than \( d_{\text{min}} \) (\( d_{\text{min}} \) must be the smallest side of the compressive member) or slightly larger than a standard 2x2.

What would have happened if we were to have chosen a lower slenderness ratio? We would still find a piece of lumber that would support the load; however it would have been wasteful. Let us investigate a slenderness ratio of 20.

\[
\text{slenderness ratio} = \frac{KL}{d_{\text{min}}} \\
20 = \frac{(2)(24)}{d_{\text{min}}} \quad d_{\text{min}} = 2.4\text{in}
\]

\[
f_c = \frac{0.3E}{(\frac{KL}{d_{\text{min}}})^2}
\]

\[
f_c = \frac{(0.3)(1760000\text{psi}^2)}{30^2} \quad f_c = 1320\text{psi}
\]

\[A = \frac{P}{f_c}\]

\[A = \frac{750\text{lbs.}}{587\text{psi}^2} \quad A = 0.57\text{in}^2
\]

Now when we calculate what the dimension of the other side needs to be we find

\[A = (d_{\text{min}})(d)\]

\[0.57\text{in}^2 = (2.4\text{in})(d) \quad d = 0.24\text{in.}
\]

\( d \) cannot be 0.24 in. because \( d_{\text{min}} \) must be the dimension of the smallest side. Therefore in order to make a slenderness ratio of 20 work, the piece of wood would have to be 2.4 in. on both sides which has an area of 5.76 in\(^2\) which is much larger than the required area for a slenderness ratio of 30.

One thing that must be remembered is that this is theory. You could cut a piece of 4 x 4 down to 1.6 in. x 1.6 in. or you could try different slenderness ratios to find a more readily available (although great cross sectional area) piece of lumber to be used. Also you must always keep in mind the practicality of the material you choose. It is going to be difficult to get two bolts into a piece of lumber only 1.6 in. wide. Also just because the numbers tell you the member will not fail does not mean that the structure will not shake so much as to be distracting during a performance. Cross bracing helps stop this wiggling as much as it does to reduce the unsupported length of the column. You should never throw away standard scenic construction techniques just because the math says it will work. It is never a bad idea to perform a full-scale test on a new structure.
Review and Summary

Direct Stress Formula

\[ A = \frac{P}{f} \quad (\text{Eq.1}) \] where \( A \) = area, \( P \) = load, and \( f \) = allowable unit stress

Radius of Gyration

The geometric property of a cross section used to determine resistance to buckling in a column

Slenderness ratio

\[ \text{slenderness ratio} = \frac{Kl}{r_{\text{min}}} \quad (\text{Eq.2}) \] where \( l \) = the unsupported length of the column and \( r_{\text{min}} \) = the minimum radius of gyration and \( K \) = constant depending upon the end conditions.

\[ \text{slenderness ratio for rectangular solid} = \frac{Kl\sqrt{12}}{d_{\text{min}}} \] where \( d_{\text{min}} \) = minimum width of the column.

Allowable unit stress in compression

for solid cross sections (lumber)

\[ f_c = \frac{0.3E}{\left( \frac{Kl}{d_{\text{min}}} \right)^2} \quad (\text{Eq.6}) \] where \( E \) = modulus of elasticity

Slenderness ratio limits

\[ \frac{Kl}{r_{\text{min}}} \leq 200 \quad \text{for steel} \quad \frac{Kl\sqrt{12}}{d_{\text{min}}} \leq 50 \quad \text{for wood} \]
Exercises and Problems

5 - 1 Tensional structural members

1. A piece of 1 x 1 11 ga. Box tube is used to hang a piece of scenery. Ignoring the attachment points, how much can the scenery weigh before failure is likely to occur? See Appendix B, table 4 for information about box tube.

2. A 3500 lb. load has to be supported from above (tension). What cross sectional area of steel is necessary to support this load? What cross sectional area of aluminum is necessary? For each of these cases, choose the most cost effective (lightest) structural shape to do the job.

5 - 2 Compressive structural members

3. A leg of a platform is bolted to the platform and is toe nailed to the stage floor. The leg has a load of 500 lb. on it and is 2’-0” long. What size lumber is necessary to support this load without buckling? What size steel? What size aluminum?

4. Figure 2 shows a loading diagram for a boom arm. Points A and C are securely fastened to the wall. The weight \( W = 8500 \text{ lb.} \) AB is 15’-0” long and BC = 19’-6” long. Design member AB and BC, using steel, to be able to resist the load. Consult Appendix B to help you choose appropriate size members.

![Figure 2. Problem 4](image-url)
Chapter 6  Truss Analysis

A truss is a very simple yet highly effective device used to transfer loads back to some form of support, the same way a beam does. It is, however, more efficient and lighter than an equivalent beam carrying the same load over the same span. The structure consists of straight, two-force members (compressive or tensile). Forces distributed through a truss are assumed to always be transmitted axially, that is to say the line of action of the force is parallel to the axis of the structural member involved. Members usually lie in one plane (although not always) and are joined to other members to form triangles. Joints are assumed to be pinned or frictionless hinges, even though this is not entirely true.

Trusses are used regularly in architecture because of their cost per strength ratio and they are becoming more and more common in theatrical venues. Lightweight aluminum trusses are very commonly seen in touring productions to hang lighting instruments or soft goods. Truss frames are also commonly used to stretch soft goods on, or used as lightweight scenic elements.

One important thing that must be kept in mind (although it may not make much sense at this point in your reading) is that the following discussion will concentrate on the analysis of purely planer trusses. It is usually helpful to build a box truss, which is simply two planer trusses with horizontal supports connecting each planer truss. In this way the unsupported length of the truss cord members is greatly reduced. This will be discussed in greater detail later in this chapter.

6-1 What is a Truss?

In chapter 4, figure 5 we had the beginning of a simple truss (reproduced here in figure 1). Member AB is completely horizontal and therefore provides a purely horizontal force. AB can do nothing to hold up the load W because all members of the truss must be either in tension or compression and act axially. W must be supported solely by member BC. All AB does is to keep the load W extended from the wall. Now what happens if we were to add additional members as in figure 2. Member CD is also a purely horizontal member and therefore...
supplies only horizontal forces. Now BD and BC are the only members capable of creating a vertical force to support the load W and AB and CD act only to keep the load extended. This example can be carried out further, but the point is this; the top and bottom members of a truss (known as the *cords*) simply keep the loads extended while the diagonal members (known as the *web members*) actually support the load. This is true for all trusses.

**Types of Common Trusses** (Simplified Truss Design p. 115-128)

**Simple Triangle**

The simplest of trusses is made of one triangle with two members in tension and one in compression. This system is limited in its span by the compressive member (see figure 3a). As the span length increases, the compressive member must get longer, and as the compressive member gets longer, it is going to have to have greater strength to resist buckling.

**Howe**

The Howe truss is characterized by the fact that the tension members between the two chords are all vertical while the angled members are in compression (see figure 3b).

**Pratt**

The Pratt is characterized by the fact that all of the compression members are vertical and all of the tensional members are angled and parallel to each other (see figure 3c).

**Warren**

The Warren truss is the most likely truss to be encountered. All members of the top and bottom chords being of equal lengths characterize this type of truss. All of the diagonal members, be they in compression or tension, also have equal length and alternate being in compression and tension (see figure 3d).
These truss classifications are named after engineers who either invented the style or who made the style famous. All of these descriptions as to diagonal members being in either compression or tension, hold true only for symmetrically loaded trusses. Non-symmetrically loaded trusses can cause some confusion as to what classification the truss falls in and many trusses become classified as “modified Warren”, or “modified Pratt”.

![Diagram of four common truss styles: Simple triangle, Howe, Pratt, Warren](image)

**Figure 3. Four common truss styles.**

### 6 - 2 Determining Forces at Work in a Truss – Method of Joints

The simplest analysis method of a truss is based on the definition of equilibrium. Recall that

\[
\begin{align*}
\sum f_v &= 0 \quad \text{Eq.1} \\
\sum f_h &= 0 \quad \text{Eq.2}
\end{align*}
\]

the sum of the forces in the vertical direction and the sum of the forces in the horizontal direction must be equal to 0. Using this definition of equilibrium and free body diagrams as our tools, we can begin to analyze truss systems. There are some assumptions that must be made in the analysis of trusses. These are as follows;

1. All members of the truss lie in the same plane.
2. Loans and reactions are applied only at the joints of the truss
3. The truss members are connected with frictionless pins (K=1)
4. All members are straight and are two-force members
5. The line of action of the internal force within each member is axial.

6. The change in length of any member due to tension or compression is not of sufficient magnitude to cause an appreciable change in the overall geometry of the truss.

7. The weight of each member is very small in comparison with the loads supported and is therefore neglected.

(Applied Statics and Strength of Materials)

These assumptions are obviously not reality. For example welded joints surely do not behave like “frictionless pins”, however these assumptions are necessary in order to mathematically analyze the structure. Experience has shown us that even though these assumptions do not represent reality we will still get useful results.

The first step in determining the force in a truss member is to find a point on the truss where there are no more than two unknown forces. Because we have two useful equations of equilibrium (\(\Sigma F_h=0\) and \(\Sigma F_y=0\)) we can only deal with two unknown variables. Draw a at this point free body diagram from the loading diagram of the truss. You will usually be able to determine whether the force is in compression or tension from visual inspection of the system. If the wrong choice is made it will become apparent in the answer by the appearance of a negative value for the force. If this does occur make sure to reverse the direction of the force in question in your free body diagram so that the sense of the force (tension or compression) will be correctly carried through later calculations.

From the free body diagram, sum the forces in the vertical direction, making sure to only take the vertical components of any forces not acting purely in the vertical direction. Set this sum equal to zero. Remember that the vector component is given by

\[
\begin{align*}
a_v &= a \sin \theta & (\text{Eq.3}) \\
a_h &= a \cos \theta & (\text{Eq.4})
\end{align*}
\]

If a vector is pointing at 90° or 270° (i.e. pointing straight up or straight down) the vector component will be equal to the magnitude of the vector with the appropriate sign attached (positive for up, negative for down). When the vector is at some angle other than 90° or 270°, the vertical vector component will be the magnitude of the vector times the sine of the angle. This will enable you to solve for one of the unknown forces. Repeat this procedure for the horizontal forces to solve for the other unknown force. Remember that when the vector is pointing at 0° or 180° (i.e. pointing straight to the left or straight to the right) the vector component will be equal to the magnitude of the vector with the appropriate sign attached (positive for right, negative for left). Again, when the vector is at some angle other than 0° or 180° the vector component will be the magnitude of the vector times the cos of the angle. Note that if you use the full 360° coordinate system the signs will always work themselves out. For example, the sine of 270° is negative 1 and when this is multiplied by the magnitude of the vector, you simple get the magnitude of the vector back with a negative sign.
With these two forces solved for, it will open up adjacent truss members to be solved by reducing the number of unknown forces at these adjacent points. This procedure is repeated until you reach the half way point of the truss (for uniformly distributed loads) or until the opposite end of the truss is reached. It may be time consuming, but this method is guaranteed to work for all trusses.

Sample problem 1

We have a 6 foot by 2 foot Warren truss with diagonal members at 45°. There are 200 lb. loads attached at each bottom joint and it is supported from two points attached at the two upper, outside joints (see figure 4). Find the stresses in all members.

![Figure 4. Sample problem 1 loading diagram](image)
It will be helpful to solve for the unknown reactions at the very beginning of the problem. Because this truss is symmetrically loaded (later we will learn how to solve for a non-symmetrically loaded truss) each reaction will support half of the load. The total load is 1000 lb. (for now we will ignore the weight of the truss) which makes each reaction equal to 500 lb.

There are two places where there are only two unknown forces, A and I. Let’s start at A and draw a free body diagram. I am making the assumption that AB is in tension and AC is in compression. It becomes intuitive with practice. We can tell from the free body diagram that this is the case from the direction that vectors are pointing. AB is pulling away from the point in question and AC is pushing toward the point in question. From this point we can write two equations from the definition of equilibrium

\[
\sum f_y = 0 = 200 \sin 270 + AB \sin 45^\circ
\]

\[
0 = -200 + AB \sin 45
\]

\[
200 = AB \sin 45
\]

\[
200 = AB(0.7071)
\]

\[
282.8 \text{ lbs. of tension} = AB
\]

Notice that we can eliminate a step if we see at the start that the 200 lbs. is a negative vertical force and simply skip to the second line. However, for now it is a good idea to continue multiplying even pure vertical forces by the sine of the angle they make.

Now we can perform a similar operation with the horizontal forces.

\[
\sum f_x = 0 = AC \cos 180^\circ + AB \cos 45^\circ
\]

\[
0 = -AC + 282.8 \cos 45^\circ
\]

\[
AC = 282.8(0.7071)
\]

\[
AC = 200 \text{ lbs. of compression.}
\]

Here we take the value for the magnitude of AB from where we solved for it above and replace AB with its numerical value.
We now know the forces at work in the first two members of the truss. Now we move to the next point that has only two unknown forces. Point B looks good (not to mention that it is in alphabetical order). Let's look at a free body diagram at point B.

Notice that in this free body diagram that $AB$ is pointing in the opposite direction as it did in figure 5, however it still has the same sense, it is still in tension (pulling away from the point in question). $BD$ we will assume to be in compression due to the nature of a symmetrically loaded Warren truss. $BC$ is assumed to be in tension because if it were in compression it would be pointing up and to the left. In order to satisfy Eq. 2, there must be at least one vector pointing in each direction left and right. The sum of the horizontal forces could never be zero if all horizontal components are pointing in the same direction. Remember that these are simply educated assumptions and if the assumption is incorrect, the final answer will be found to be negative and then we can go back and fix it. Using Eq 1 we find

$$
\sum f_x = 0 = 500 \sin 90^\circ + AB \sin 225^\circ + BC \sin 315^\circ \\
0 = 500 + 282.8 \sin 225^\circ + BC \sin 315^\circ \\
0 = 500 + 282.8(-0.7071) + BC(-0.7071) \\
0 = 500 - 200 - 0.7071 BC \\
0 = 300 - 0.7071 BC \\
0.7071 BC = 300 \\
BC = 424.3 \text{lb. of tension}
$$

and using Eq.2 we find

$$
\sum f_y = 0 = BD \cos 180 + AB \cos 225 + BC \cos 315 \\
0 = BD(-1) + 282.8 \cos 225 + 424.3 \cos 315 \\
0 = -BD + 282.8(-0.7071) + 424.3(0.7071) \\
BD = -200 + 300 \\
BD = 100 \text{ lb. of compression}
$$

Let's look at one final free body diagram at point C.
Again start with Eq.1

\[ \sum f_x = 0 = 200 \sin 270 + BC \sin 135 + CD \sin 225 \]
\[ 0 = 200(-1) + 424.3(0.7071) + CD(-0.7071) \]
\[ 0 = -200 + 300 \cdot 0.7071CD \]
\[ 0.7071CD = 100 \]
\[ CD = 141.4 \text{ lb. compression} \]

and from Eq.2

\[ \sum f_y = 0 = AC \cos 0 + CE \cos 0 + BC \cos 135 + CD \cos 225 \]
\[ 0 = 200 + CE(1) + 424.3(-0.7071) + 141.4(-0.7071) \]
\[ 0 = 200 + CE - 300 - 100 \]
\[ CE = 200 \text{ lb. Tension} \]

With this half of the truss members solved, we can see that the opposite half will be a mirror image of the first half. In the case of a symmetrically loaded truss, you only need to solve for half of the truss members because the opposite half will be the same.

When forces are determined for all truss members, you can now use the methods developed in chapter 5 to determine what materials should be used to resist the given forces. You will usually find that the members in compression will have to be the strongest of the truss. This is because they will have to resist buckling whereas the tensional members only have to resist being pulled apart. It is reasonable to think that there will be many different materials composed in the truss. You could use cable for the tensional members, large box tube for the high compression (or long length) members, small angle iron for the lower compression (or shorter length) members, for example, but that would make for a strange looking and difficult to build truss. It is much more efficient to calculate the required material for the worst case in compression and the worst case in tension and use appropriate material for each. In some cases you might wish to calculate the overall worst case and build the entire truss from the one material, or solve for the worst case in the cords and the worst case in the webs and build the truss members out of the appropriate materials.

6 - 3 Finding reactions of a non-symmetrically loaded truss

We can see from sample problem 1, that the value of the reactions is necessary to solve for the forces in the members of the truss. In sample problem 1, finding the reactions was a simple matter due to the symmetric nature of the given truss. However, what can we do for a non-symmetrically loaded truss? To discover reactions in this case, we need to study the moments acting on the truss.
A moment is defined as the tendency of a force to cause rotation about a point or axis. In an equilibrium system, the sum of the moments must sum to zero just as the sum of the vertical and horizontal forces must be zero. The mathematical definition of a moment is

\[ M_a = Fd \quad (\text{Eq.5}) \]

where \( M_a \) = the moment at pivot point a
\( F \) = the force applied at the point in question
\( d \) = the perpendicular distance from where the force is applied to the pivot point

To use moments to solve for unknown reactions in non-symmetrically loaded truss, choose one reaction to be the pivot point. Then sum the moments by multiplying the force of the first load by the distance that the load is from the reaction and adding it to the product of the next load and the distance it is from the reaction, etc. Then using the definition of equilibrium

\[ \sum M_a = 0 \quad (\text{Eq.6}) \]

you can solve for the value of the other reaction. Remember the accepted sign convention. Any force which tends to cause a clockwise rotation is given a negative sign. Any force that tends to produce a counter-clockwise rotation is given a positive sign. The mathematics will work even with this convention reversed, however it is a good idea to stay with the consensus.

Sample problem 2.

Given the truss in figure 8, solve for \( R_1 \) and \( R_2 \).

First we need to decide which reaction to begin with. It really does not matter so lets begin with \( R_1 \). Now we can set up the equilibrium equation from Eq.6.

\[ \sum M_a = 0 = -400(1) + 200(1) + 300(3) + 800(5) - R_2 \]

\[ 0 = -400 + 200 + 900 + 4000 - 4R_2 \]

\[ 4R_2 = 4700 \]

\[ R_2 = 1175 \text{ lbs.} \]
The 400 lbs. force and R2 are both negative because they tend to produce counter clockwise rotation. To solve for R1, simply sum the moments about R2.

\[
\sum M_{R2} = 0 = 400(5) - R_1(4) + 200(3) + 300(1) - 800(1) \\
0 = 2100 - 4R_1 \\
4R_1 = 2100 \\
R_1 = 525 \text{ lbs}
\]

As a check it is a good idea to use \( \Sigma F_v = 0 \) to see if the sum of the vertical forces truly do sum to 0

\[
\sum F_v = 0 = -400 + 1175 - 200 - 300 + 525 - 800 \\
0 = 0 \quad \text{Check}
\]

Now that we know the value of both the reactions, the remaining analysis of the truss follows just as it did in sample problem 1.

6 - 4 Method of Sections

The method of joints can be a very long and arduous process for a very long truss. The Method of Sections is a technique that is useful when only a few truss members need analyzing. For example, after looking closely at a symmetrically loaded truss, we find that the largest compression force in the web members will be near the reactions and the largest compression force in the chords will be at the center of the truss. It is therefore not necessary to solve for each and every member of the truss if you can be sure of where the worst cases in compression are located. The key to this method is to choose the correct section of the truss to examine. Finding the value of the compression force in a member that is not the worst case will do little good.

The method of sections is accomplished by ‘cutting’ the truss through the members that have been discerned to be the worst case. This cutting is purely an academic process for the purposes of examining the truss as a free body diagram. To employ the Method of Sections you pass a plane through three members, one of which is the member you are looking to analyze. Discard either the left hand or right hand portion of the truss. Then, by choosing the intersection of the two members which aren’t being analyzed from the remaining section as the moment center, apply the equilibrium equations \( \Sigma F_v = 0, \Sigma F_H = 0 \) and \( \Sigma M = 0 \) to all remaining external forces, reactions and members cut. Be sure to ONLY use external forces, reactions and cut members from the section of the truss remaining. The following sample problem will explain this process in more detail.
Sample problem 3

Solve for center chords and web members in the truss shown in figure 9.

The first step is to cut the truss through the section under consideration. This area is marked with the dotted line X-X in figure 9. This plane was chosen because it cuts through the chord and web members we are interested in. Next draw a free body diagram showing the remainder of the truss after the cut with the cut chords and web replaced with force vectors. See figure 10.

Figure 9. Loading diagram of truss for sample problem 3. Dotted line X-X represents cutting line.

Figure 10. Free body diagram of truss for sample problem 3.
Notice that the three unknown forces, P1, P2 and P3 are all drawn in tension. If, after the values for these forces are determined, the answers are shown to have a negative value then we know that the force is actually in compression with the same magnitude.

Next, the definition of equilibrium in relation to moments is applied to the free body diagram. The summation of moments must be in relation to a point where there is only one unknown. The only place that this occurs in this problem is at point A, where two of the three forces intersect.

\[
\sum M_A = 0 = 500(6) - 200(5) - 200(3) - 200(1) + P_1(2)
\]
\[
0 = 3000 - 1000 - 600 - 200 + P_1(2)
\]
\[
-1200 = P_1(2)
\]
\[
P_1 = -600 \text{ lb.}
\]

Because \(P_1\) has a negative value, the force must be in compression instead of tension as we drew it. Now because \(P_2\) is the only unknown force with a vertical component, and the sum of the vertical forces must be zero

\[
\sum F_v = 0 = 500 - 200 - 200 - 200 + P_2 \sin 315
\]
\[
0 = -100 + P_2 \sin 315
\]
\[
100 = P_2 \sin 315
\]
\[
P_2 = -141.42 \text{ lb.}
\]

Again, the value has been shown to have a negative value, therefore \(P_2\) is also in compression. The last unknown force, \(P_3\), can be found by summing the moments around point B

\[
\sum M_B = 0 = 500(5) - 200(4) - 200(2) - P_3(1)
\]
\[
0 = 1300 - P_3
\]
\[
P_3 = 1300 \text{ lb.}
\]

\(P_3\) came out having a positive value and therefore it is in tension as was drawn.

6 - 5 Material Choices

Because all members of a truss are either in compression or in tension, the same processes described in chapter 5 can be used to design the webs and cords of a truss.

Tension members

The direct stress formula is used to design members of the truss found to be in tension. Remember the direct stress formula is
\[
A = \frac{P}{f_t} \quad \text{ (Eq.1) }
\]

where \( A \) = the required cross-sectional area of the tensional member (in\(^2\))
\( P \) = the load placed on the tensional member (lbs.)
\( f_t \) = the allowable unit stress in tension for the material used (\( \frac{\text{lbs}}{\text{in}^2} \))

This is used precisely the same way you used it in chapter 5.

**Compression members**

The design of the compression members of the truss follows the same procedure as designing a compressive column. Based upon the unsupported length of the member and the force applied to the member, you can choose an appropriate slenderness ratio. Once the slenderness ratio has been chosen, you can calculate the value for the minimum radius of gyration or the minimum depth of the compression member using the slenderness ratio formulas from chapter 5.

\[
\frac{Kl}{r_{\text{min}}} = \text{slenderness ratio} \quad \text{or} \quad \frac{Kl\sqrt{12}}{d_{\text{min}}} = \text{slenderness ratio} \quad (\text{Eq.2})
\]

In addition to the minimum radius of gyration or minimum depth, the decision of slenderness ratio leads to the allowable unit stress in compression (\( f_c \)). This is either found in Appendix B, table 1 for steel, table 2 for aluminum or by using the following for lumber;

\[
f_c = \frac{0.3E}{\left( \frac{Kl}{d_{\text{min}}} \right)^2} \quad (\text{Eq.3})
\]

For more information on the use of these formulas, review chapter 5.
Review and Summary

Definition of Equilibrium

\[ \sum F_v = 0 \]
\[ \sum F_h = 0 \]
\[ \sum M = 0 \]

Vector components

\[ a_v = a \sin \theta \]
\[ a_h = a \cos \theta \]

where \( a_v \) = vertical component of vector \( a \) and \( a_h \) = the horizontal component of vector \( a \).

Moment

\[ M = fD \]

where \( M \) = the moment about a given pivot point, \( f \) = the force applied and \( D \) = the distance the force is applied from the pivot point.

Method of Joints

Begin at a point where there are two or less unknown forces and draw a free body diagram at that point. This will allow you to solve for two forces involved in that free body diagram which, in turn, will allow you to draw free body diagrams at an adjacent point. For symmetrically loaded trusses, it is only necessary to solve out to the center of the truss.

Method of Sections

Pass a plane through three members of the truss. Using the definition of equilibrium you can solve for the web and chord members cut through. This method allows you to go directly to the crucial part of the truss without first having to solve for every structural member.
Exercises and Problems

6 - 2 Determining forces at work in a truss
1. Given the truss in figure 11, and W = 500 lb., calculate the forces at work in each reaction and in the structural members.
2. Given the truss in figure 11, and W = 1150 lb., calculate the forces at work in each reaction and in the structural members.

![Figure 11. Truss loading diagram](image)

3. From your solutions to problem 1, what steel structural shapes are necessary for this truss? What about aluminum? Remember that the worse case is usually (but not always) in the compressive members.

6 - 2a Finding reactions from a non-symmetrically loaded truss
4. Find the reactions of the truss in figure 11 with the following loads:
   - \(W_1 = 500 \text{ lb.} \theta = 45^\circ \ L = 2\text{-}0''\)
   - \(W_2 = 300 \text{ lb.}\)
   - \(W_3 = 750 \text{ lb.}\)
   - \(W_4 = 900 \text{ lb.}\)
   - \(W_5 = 125 \text{ lb.} \theta = 45^\circ \ L = 3\text{-}0''\)

5. Find the reactions of the truss in figure 11 with the following loads:
   - \(W_1 = 225 \text{ lb.} \theta = 45^\circ \ L = 3\text{-}0''\)
   - \(W_2 = 900 \text{ lb.}\)
   - \(W_3 = 550 \text{ lb.}\)
   - \(W_4 = 210 \text{ lb.}\)
6. Using the method of sections and the truss in figure 11 with the following loads, determine the forces at work in all cord and web members. What steel shapes will support the loads?

- \( W_1 = 400 \text{ lb.} \), \( \theta = 60^\circ \), \( L = 2' - 6'' \)
- \( W_2 = 500 \text{ lb.} \)
- \( W_3 = 100 \text{ lb.} \)
- \( W_4 = 300 \text{ lb.} \)
- \( W_5 = 400 \text{ lb.} \)

7. Using the method of sections and the truss in figure 11 with \( W = 225 \text{ lb.} \), \( \theta = 60^\circ \) and \( L = 2' - 0'' \) determine the forces at work in all cord and web members. What aluminum shapes will support the loads?
Chapter 7  Design of Beams

7 - 1 Types of Beams

Beams are among the most common structural members. The frame of a platform and a pipe batten are both examples of beams commonly found in theatrical applications. A beam is a structural member used to resist transverse loads. The supports for beams are usually near the ends and apply their resistive forces (reactions). The loads acting on a beam tend to bend the member, as opposed to lengthening or shorting the member (as in a truss). There is a subtle but important difference between a beam and a truss. First of all, a truss is assumed to not deflect where as a beam develops its resistance to the load by bending. As a load is applied to a beam it bends, which sends the fibers or molecules on one side of the beam into compression and the fibers or molecules on the opposite side into tension. It is the ability of these fibers to resist these compressive and tensional forces that allows the beam to resist the load. However, these compression and tensional forces are not equally distributed across the cross section of the beam. The greatest force occurs near the outside of the beam and it decreases as you approach the center of the beam, until you reach the centerline of the beam where all compression and tension forces reach zero. Therefore it is the characteristics of the cross section of the beam that determines its ability to resist bending.

There are, in general, five basic types of beams, which are described by the number of and position of supports. They are described here (see Figure 1). (Simplified Mechanics and Strength of Materials p.125 -126)

A simple beam rests on a support on each end, with the ends free to rotate. A board placed on top of two blocks is a good example.

A cantilever beam is supported on one end only. This one end is fixed in rotation and translation. A beam embedded in a concrete wall is a typical example.

An overhanging beam is one in which the end (or ends) project over the support. The overhanging beam is commonly mislabeled as a cantilever.

A continuous beam rests on more than two supports. The side of a 4 x 8 platform with three legs is an example of a continuous beam.

A restrained beam has both ends fixed in translation and rotation. A beam embedded in a wall on both sides is an example of a restrained beam.

The beam type is determined by how its ends are supported. There are two types of end supports, the Pin and the Roller (see Figure 2). The roller end support will allow the beam to slide left and right, but will not allow it to move up or down (it will only supply a vertical reaction). The Pin end support will prevent the beam from sliding left and right in addition to preventing it from moving up and down (it will supply both vertical and horizontal reactions).
7 - 2 Beam Loads

There are two types of loads commonly supported by a beam, concentrated and distributed. A *concentrated* load is when the load acts at a single point of a beam (or at least a very small area when compared to the length of the beam). This, of course, is impossible in reality, but the assumption does work. A concentrated load could be a column being supported by a beam, for example. A *distributed* load is when the load is spread out over a large length of the beam. A row of bricks lined up end to end along a beam is an example of a distributed load. There will be times when a number of closely spaced concentrated loads will be calculated as if they were a distributed load and it is possible to convert a distributed load into an equivalent concentrated load. The total load acting at the midpoint of its length can represent a distributed load. Both of these cases will be discussed later in this chapter. A sub category of the distributed load is the *uniformly distributed load* in which the load applies a constant magnitude of force per linear unit of the length of the beam.

7 - 3 Determining Beam Reactions

A reaction is the upward force supplied by the supports of the beam that resist the loads. If there is only one load, centered on a beam, and two supports located at either end of the beam, it is intuitive to see that each support will hold half the load. However, this case is somewhat rare. What we need to do in most cases is study the moments acting on the beam in a way similar to the process we used for the case of non-symmetrically loaded trusses.
Sample problem 1

There is a 10'-0" beam with supports at each end and two loads placed on it. The first load is 2'-0" from one side and the second load is 6'-0" from the first load (see figure 1). Find the value of the two reactions.

To determine the reactions first we need to sum the moments around one reaction and set that sum equal to zero.

Remember that a moment is defined as the force multiplied by the distance the force is from the point being examined.

\[ M = fd \]

\[
\sum M_{R_1} = 0 = -(400)(2) - (750)(8) + R_2 \cdot 10
\]

\[ 6800 = 10R_2 \]

\[ R_2 = 680 \text{ lbs} \]

\[
\sum M_{R_2} = 0 = -R_1(10) + 400(8) + 750(2)
\]

\[ 10R_1 = 4700 \]

\[ R_1 = 470 \text{ lbs} \]

To check our answer, use the equilibrium \( \Sigma F_y = 0 \)

\[
\sum F_y = 0 = 470 - 400 - 750 + 680
\]

\[ 0 = 0 \quad \text{Check} \]

When dealing with a uniformly distributed load, it is necessary to convert it to an equivalent point load to calculate the moments used in determining the reactions. For a uniformly distributed load, the equivalent point load will be the total distributed load applied at the center of the uniformly distributed load. A non-uniformly distributed load can usually be broken into several uniformly distributed loads, and equivalent point loads applied.
7 - 4 Vertical Shear in Beams

There are generally two separate types of shear in beams. They are vertical shear (denoted with a capital V) and horizontal shear (denoted with a lower case v). Vertical shear is defined as the tendency for one part of a member to move vertically with respect to the adjacent part of the member, as shown in figure1.

If we refer to the entire uniformly distributed load acting on the beam in figure 2 as W, and we assume that the loading is symmetrical, then we can say that the value for each of the reactions is W/2. If we look at just the left support we see that the support is pushing up (positive) on the beam with a value of W/2. Just to the right of the support the load is pushing downward (negative) and we can say that the magnitude of the tendency of the left and right portions to slide past each other (the vertical shear) is equal to the value of the reaction, W/2. As we move out over the beam to the right, the value of the distributed load increases (because we are including more of it in our study area) and that subtracts from the shear force by an amount equal to the value of the load. The magnitude of the vertical shear at any section of a beam is equal to the algebraic sum of all vertical forces on one side of the section. Written mathematically

\[ V_x = \sum R - \sum L_x \quad \text{(Eq.1)} \]

where
- \( V \) = the vertical shear (lbs.)
- \( R \) = the reaction to the left of the study area (lbs.)
- \( L_x \) = the load at point x (lbs.)

When designing beams it is usually useful to draw a shear diagram showing visually the value of the vertical shear at all points along the beam. These shear diagrams are simply a graph with the distance along the beam on the X axis and the value of the vertical shear along the Y axis.
Sample problem 2

Using the same beam from sample problem 1, calculate the values of vertical shear and show the corresponding shear diagram.

Start with the expression for vertical shear from Eq.1

\[ V_s = R - \sum L_s \]

- \[ V_{(x=1)} = 470 - 0 = 470 \text{ lbs.} \]
- \[ V_{(x=2)} = 470 - 400 = 70 \text{ lbs.} \]
- \[ V_{(x=3)} = 470 - 400 = 70 \text{ lbs.} \]
- \[ V_{(x=4)} = 470 - 400 - 750 = -680 \text{ lbs.} \]
- \[ V_{(x=5)} = 470 - 400 - 750 = -680 \text{ lbs.} \]

The shear diagram would look like the following

The procedure is very similar for a system that has a beam with a uniformly distributed load.

Sample problem 3.

Find the vertical shear and draw the shear diagram for a 12'-0" beam with an equally distributed load on it of 50 lbs per linear foot.
The procedure is just the same as it is in a beam with point loads as in sample problem 2.

In order to calculate the shear force acting on this beam, we need to determine the value of the reactions. To calculate the total load, simply multiply the distributed load by the length of the beam.

\[
\text{Total load} = (50 \text{ lbs/ft})(12 \text{ ft}) = 600 \text{ lbs}.
\]

Because the load is equally distributed across the length of the beam, the sum of the reactions must be equal to the total load and both reactions must be equal to each other.

\[
\text{Total load} = R_1 + R_2
\]
\[
R_1 = R_2
\]
\[
\text{Total load} = 600 \text{ lbs.} = 2R_1
\]
\[
R_1 = 300 \text{ lbs.}
\]

And because \( R_1 = R_2 \),

\[
R_2 = 300 \text{ lbs.}
\]

Now that we know the reactions, we can use Eq.1, to calculate the values for shear at various points along the beam

\[
V_x = R - \sum L_x
\]
\[
V_{(x=0)} = 300 - (0 \cdot 50) = 300 \text{ lbs.}
\]
\[
V_{(x=1)} = 300 - (50 \cdot 1) = 250 \text{ lbs.}
\]
\[
V_{(x=2)} = 300 - (50 \cdot 2) = 200 \text{ lbs.}
\]
\[
V_{(x=6)} = 300 - (50 \cdot 6) = 0 \text{ lbs.}
\]
\[
V_{(x=7)} = 300 - (50 \cdot 7) = -50 \text{ lbs.}
\]
\[
V_{(x=12)} = 300 - (50 \cdot 12) = -300 \text{ lbs.}
\]

Note that in this case, because of the uniformly distributed load, the sum of the loads is equal to the load per foot given in the problem, multiplied by the distance from the end of the beam. With these numbers, we can draw a shear diagram to visually describe the shear trend in this beam (see figure 5.)
From figure 5, we can very easily see that the shear starts at a maximum at the left reaction, and steadily decreases to zero at the center point of the beam, and continues to decrease to a maximum negative value at the right reaction.

7 - 5 Horizontal Shear in Beams

Imagine a number of boards, stacked flat, one upon another, with a load placed on top and two supports, one on each side, underneath. The load will tend to cause the boards to slip past one another (see figure 6). There is the same tendency in solid beams. The fibers tend to slide horizontally. This is known as horizontal shear.

At any point in a beam, the horizontal shear is directly proportional to the vertical shear. However, the value of horizontal shear is not evenly distributed throughout the beam. Horizontal shear has its maximum value along the neutral axis (which is typically along the length of the center of the beam for symmetrical sections). The unit value for shearing stress can be determined using Eq.2.
\[ v = \frac{VQ}{lb} \quad \text{(Simplified Engineering for Architects and Builders) (Eq.2)} \]

where \( v \) = unit horizontal shearing stress (lb/in)

\( V \) = total vertical shear at the selected section (lbs.)

\( Q \) = the statical moment with respect to the neutral axis of the area above the point at which \( v \) is to be determined (in\(^3\))

\( I \) = The moment of inertia of the cross section of the beam (in\(^4\))

\( b \) = The width of the beam at the point at which \( v \) is to be computed (in.)

Eq.2 can be simplified for the most common cases of beams. Wooden beams are almost always of a solid square cross section. If we replace certain expressions in Eq.2 with simplifications based on solid cross sections, we will have a much more useful equation. In the case for rectangular beams, the maximum horizontal shear is going to occur at the neutral axis and it is the maximum shear that we are primarily concerned with. The area above the neutral axis is \((b = d / 2)\), and its centroid is \(d / 4\) units from the neutral axis. Therefore

\[ Q = b \cdot \frac{d}{2} \cdot \frac{d}{4} = \frac{bd^2}{8} \]

The moment of inertia, \( I \), of the cross section is equal to \(bd^3/12\) (see appendix C, table 2). Therefore

\[ v = \frac{VQ}{lb} = \frac{V(bd^2 / 8)}{(bd^3 / 12) b} = \frac{3V}{2bd} \quad \text{(Simplified Engineering for Architects and Builders) (Eq.3 For solid rectangular beams)} \]

Sample problem 4

A simply supported Douglas fir 2 X 4 with an 8'-0" span has a uniformly distributed load (including the weight of the beam) of 50 lb./ft over the entire length. Compute the maximum horizontal shear.

To calculate the reactions, treat the distributed load as the total load applied at the center of the distributed length. In this case it is 400 lb. applied 4'-0" from the left (or right) side of the beam. Then use the definition of equilibrium as applied to moments.

\[ \sum M_{R_i} = 0 = -(400)(4) + R_2(8) \]
\[ 8R_2 = 1600 \]
\[ R_2 = 200\text{lbs} \]

\[ \sum M_{R_i} = 0 = -R_1(8) + 400(4) \]
\[ 8R_1 = 1600 \]
\[ R_1 = 200\text{lbs} \]

Just as we would expect.
Next, using Eq.1

\[ V_i = R - \sum_{i=0}^{n} L_i \]

\[ V_{(x=0)} = 200 - (50 \cdot 0) = 200 \text{ lbs.} \]
\[ V_{(x=1)} = 200 - (50 \cdot 1) = 150 \text{ lbs.} \]
\[ V_{(x=5)} = 200 - (50 \cdot 5) = -50 \text{ lbs.} \]
\[ V_{(x=8)} = 200 - (50 \cdot 8) = -200 \text{ lbs.} \]

It is also sometimes helpful to find the point where vertical shear crosses zero.

\[ V = 0 = 200 - (50 \cdot X) \]
\[ 50X = 200 \]
\[ X = 4 \text{ ft} \]

From Eq.3 we can calculate the horizontal shear. We are only concerned with the maximum value of horizontal shear and therefore, we take the largest value from \( V \) (vertical shear calculated above) for our calculations in Eq.3

\[ v_{\text{max}} = \frac{3V_{\text{max}}}{2bd} \]
\[ v = \frac{3(200)}{2(1.5)(3.5)} = 57.1 \text{ lbs.} \]

So far we have only discussed the process for designing solid rectangular beams. Usually steel beams will not be solid nor will they be rectangular. The assumptions taken to produce Eq.3 no longer apply, however, the maximum horizontal shear will still occur at the neutral axis and the horizontal shear at the extreme fibers will be zero for wide flange shapes and I-beams. With horizontal shear essentially zero at the extreme fibers we can assume that the flanges have little influence on resistance to horizontal shear and therefore they can be ignored and only the web portion be considered. Based on this assumption the following expression can be employed;

\[
 f_v = \frac{V}{dt_w} = \frac{V}{A_w} \quad \text{(Simplified Mechanics and Strength of Materials)} \\
 \text{(Eq.4 for wide flange and I-beam steel sections)}
\]

where \( f_v \) = the shearing stress (lb/in\(^2\)),
\( V_{\text{max}} \) = the maximum vertical shear (lbs.),
\( d \) = the overall depth of the beam (in.),
\( t_w \) = the thickness of the beam web (in.),
\( A_w \) = the total area of the web (in\(^2\))

The maximum allowable horizontal shear is given by the following equation;
Sample problem 5

A 3 in. I-beam is determined to have a maximum vertical shear force applied to it of 10,000 lb. The I-beam has a web thickness of 0.25 in. Find the value for shearing stress $f_v$. Will the steel I-beam withstand this shear force?

$$f_v = \frac{V}{d_t} = \frac{10000}{(3)(0.25)} = 13,333 \text{ lb.}$$

To find out whether the beam can handle this load we apply Eq.5

$$f_{v\text{max}} = 0.40 f_y$$

$$f_{v\text{max}} = 0.40(36000) = 14,400 \text{ lb.}$$

Because $f_{v\text{max}}$ is greater than the actual shear $f_v$, the answer is yes, the beam will not fail due to shear.

### 7 - 6 Bending Moments in Beams

The moment of a force is the tendency of that force to produce rotation about a certain point. This point is called the center of moments, and the perpendicular distance between the center of moments and the point at which the force is applied is known as the moment arm. Remember that a moment that tends to produce clockwise rotation is designated with a negative sign and a moment which tends to produce a counter-clockwise rotation is designated with a positive sign.

A beam bends because of the forces applied to it by the loads and the reactions. Consider point x in figure 7. The reaction, $R_1$, tends to cause a clockwise rotation about point x. The moment at this point will be the magnitude of the force multiplied by the moment arm. In this case the value of the force is 2000 lbs. (calculated using methods discussed in section 7 - 4) and the moment arm distance is 6 feet, giving the moment at point x a value of 12,000 ft-lbs and tends to cause clockwise rotation. This value could also be arrived at by studying the forces to the right of point x. There are two forces, the load L (8,000 lb.) and $R_2$ (6,000 lbs). The moment due to the reaction is $10 \times 6000 = 60,000$ ft-lbs. and it tends to cause counter-clockwise rotation about point x (which you should remember is denoted with a positive sign). The moment due to the load is $8000 \times 6 = 48,000$ lbs and it tends to cause clockwise rotation (which carries with it a negative sign). The resultant moment is the sum of the two moments or $60,000 - 48,000 = 12,000$ ft-lbs. causing counter-clockwise rotation. Notice that this is the same magnitude with an opposite direction. Therefore we can see that it makes no difference whether we
examine the forces to the left or the right, the magnitude of the moment is the same. This moment is known as the bending moment. The magnitude of the bending moment changes as you look at different sections of the beam. For example, the bending moment 4'-0" from the reaction R₁ is \(2000 \times 4 = 8000 \text{ ft-lb}\). For simplicity, let’s only look at forces to the left of the point along the beam. The bending moment at any section of a beam is equal to the moments of the reactions minus the moments of the loads to the left of the section.

\[
M_b = \sum M_i - \sum M_l
\]  \hspace{1cm} (Eq.6)

where \(M_b\) = the bending moment at a given location (ft-lbs.)

\(M_i\) = the moment of the reaction to the left of the location being analyzed (ft-lbs.)

\(M_l\) = the moment of the loads to the left of the location being analyzed (ft-lbs.)

It is sometimes useful to draw a bending moment diagram, similar to a shear diagram. A sample of moments are taken across the length of the beam and plotted on a graph with the bending moment along the Y-axis and the distance along the beam along the X-axis. The bending moment diagram is usually drawn below the shear diagram and below the loading diagram. We will find that the bending moments in a simple beam all have positive values and therefore fall above the base line. In overhanging beams and continuous beam we will find positive and negative bending moments, with the negative drawn below the line.

Sample problem 6

There is a simply supported beam, 16'-0" long, with a concentrated load of 4000 lbs. placed upon it 10'-0" from the left side. Draw a shear diagram and a moment diagram

\[
V_i = R - \sum L_i
\]

\(V_{x=0} = 1500 - 0 = 1500 \text{ lbs.}\)

\(V_{x=1} = 1500 - 0 = 1500 \text{ lbs.}\)

\(V_{x=8} = 1500 - 0 = 1500 \text{ lbs.}\)

\(V_{x=10} = 1500 - 4000 = -2500 \text{ lbs.}\)

\(V_{x=12} = 1500 - 4000 = -2500 \text{ lbs.}\)

\(V_{x=16} = 1500 - 4000 = -2500 \text{ lbs.}\)

To calculate the bending moments we use Eq.5

\[
M_b = M_r - M_l
\]

\(M_{b,x=0} = (1500)(0) - 0 = 0 \text{ ft-lbs.}\)

\(M_{b,x=1} = (1500)(1) - 0 = 1500 \text{ ft-lbs.}\)

\(M_{b,x=8} = (1500)(8) - 0 = 12000 \text{ ft-lbs.}\)

\(M_{b,x=10} = (1500)(10) - 0 = 15000 \text{ ft-lbs.}\)

\(M_{b,x=10.5} = (1500)(10.5) - (4000)(0.5) = 13750 \text{ ft-lbs.}\)

\(M_{b,x=13} = (1500)(13) - (4000)(3) = 7500 \text{ ft-lbs.}\)

\(M_{b,x=16} = (1500)(16) - (4000)(6) = 0 \text{ ft-lbs.}\)
The reactions and values for shear are computed as they were in section 7 - 4. Using Eq.1

( a ) Loading diagram

\[ R_1 = 1500 \text{ lb.} \quad R_2 = 2500 \text{ lb.} \]

\[ V = 1500 \text{ lb.} \]

( b ) Shear diagram

\[ V = 2500 \text{ lb.} \]

( c ) Moment diagram

\[ M = 15000 \text{ ft-lb.} \]
\[ M = 12000 \text{ ft-lb.} \]
\[ M = 7500 \text{ ft-lb.} \]
\[ M = 5000 \text{ ft-lb.} \]

Figure 8. ( a ) loading diagram, ( b ) shear diagram, ( c ) moment diagram
7 - 7 Deflection in Beams

Simply determining the maximum bending moment and maximum shear in a beam is usually not sufficient information to proclaim the beam size, shape and material appropriate to the job at hand. By the very nature of a beam, some deflection will occur. (In fact, it is this deflection that is responsible for forces that resist the load in a beam.) A beam might not fail in bending or in shear but it might deflect to such a point when loaded as to make people wonder if it really will resist the loads applied to it. If a platform is built and a performer stands in the center of it, it deflects so much that it looks like it is going to break, the audience could spend more time wondering if the floor is going to crash out from underneath the actors and not on the scene.

How much deflection is too much? In the architectural world, deflection is usually limited to $\frac{1}{360}$ of the total span. This number is originally derived from the maximum amount of stretch plaster ceilings could withstand without cracking. This number is not a number that must be held to in the strictest sense. The amount of deflection should be based upon the individual case in question. Simply because the deflection is limited to $\frac{1}{360}$ of the total span doesn’t guarantee a safe structure.

The general form of the equation for deflection of beams is

$$D = K \frac{P l^3}{EI}$$

(Simplified Mechanics and Strength of Materials)  

(Eq.6)

where

- $D$ = the deflection (in.)
- $P$ = the magnitude of the load (lbs.)
- $l$ = the beam (in.)
- $K$ = a coefficient related to the distribution of the load (unitless)
- $E$ = the modulus of elasticity of the material used (lbf/in$^2$)
- $I$ = moment of inertia (in$^4$)

Notice that by examining Eq.6, we see that deflection will increase proportionally with the increase of load. However, the value of deflection will increase much faster (geometrically by a factor of 3) with increasing length of the beam. We can also see that the value of deflection is inversely proportional to both the modulus of elasticity and the moment of inertia of the beam in question.

Actually deriving $K$ values to calculate the value for the deflection is, in general, a complicated process and far beyond the scope of this text. It requires the use of calculus and will not be covered. There is a somewhat simpler method known as the moment-area method that does not require calculus, although it is somewhat limited in its application. The exact nature of the moment-area method is also beyond the scope of this text, but the result of this method does provide us formulae useful in calculating the deflection for commonly loaded and supported beams. The results of these calculations are tabulated in appendix A, chart B.
7 - 8 Designing a Member to Resist Calculated Shear and Bending

In the previous sections of this chapter, we learned how to calculate the bending moment and the vertical and horizontal shear forces set up in a beam when loaded. Now we need to investigate the bending stresses set up by the beam to resist the load and we need to be able to determine if the shear in the system is greater than the allowable shear for the material chosen.

A material’s resistance to shear is determined solely by the material used. The allowable shear stress for any material is given by Eq.7.

\[ F_s = 0.4F_y \]  
(Eq.7)

For example, A36 mild steel (36,000 lb./in\(^2\)) has an allowable shear stress of 0.4 \(\times\) 36000 or 14,400 lb./in\(^2\). So long as the value for the horizontal shear is less than this value, the member will be able to resist the shear applied to it.

Designing a beam to resist bending is not as simple. Examine figure 9. The top drawing represents a beam prior to its being loaded. The top and bottom surfaces of the beam are parallel to each other. The dash-dot line represents the neutral axis of the beam. The vertical lines A-B and C-D are also parallel. The lower drawing represents the same beam with a load placed at the center of the beam. Notice how the top and bottom surfaces are now bent but are still parallel. Points A and C however have moved closer together while points B and D have moved further apart. Lines A-B and C-D are no longer parallel. The fibers above the neutral surface are now in compression and the fibers below the neutral surface have been put in tension. The maximum compressive stress is at the top surface of the beam and this compressive stress decreases as you approach the neutral surface until it reaches zero at the neutral surface. The stress then begins to increase in tension until it reaches a maximum tension at the very bottom surface of the beam.

The Flexure Formula

The compressive and tensional forces at work in a loaded beam are not uniformly distributed across the section of the beam and therefore the direct stress formula is of no use here. The expression used to calculate the value of the bending stress is known as the beam formula or the flexure formula.

\[ M_{\text{max}} = f_b S \]  
(Simplified Engineering for Architects and Builders)  
(Eq.8 - The Flexure Formula)

where \( M_{\text{max}} \) = the maximum bending moment (in-lbs.)
\( f_b \) = the allowable unit stress in bending for the given material (0.66 \( F_y \)) (lbs), found in Appendix A, table 1
\( S \) = the required section modulus (in\(^3\)), found in tables in Appendix B and C

The flexure formula involves the size and shape of the beam cross section, represented by the section modulus (S) and the material of which the beam is made, represented by \( f_b \). The value for the allowable unit stress in bending can be found in charts found in appendix A and appendix C. The value for the maximum bending moment...
is the largest bending moment as determined by the loading of the beam. The section modulus is usually the variable in the formula that is being solved for. Given values for the section modulus are found on charts and is purely a function of the shape of the section of the beam.

To determine the appropriate size, shape and material necessary to support a given load first it is necessary to calculate the maximum bending moment in the beam in question. Once the maximum bending moment has been determined, you must decide what material the beam will be constructed from. If it is steel, you must decide upon what grade of steel, keeping in mind that most commonly available structural shapes are A36 steel. If it is aluminum, what alloy and grade? If you choose wood, you need to decide upon what species and what grade. Once these decisions are made, \( f_b \) can be found on the appropriate chart and the value substituted in the flexure formula. With these two values known, it is a simple matter to solve for the section modulus and the appropriate chart consulted to find a size and shape of the beam necessary to support the load.

**Sample problem 7**

A simply supported beam has a span of 10’-0” and supports a load, including its own weight of 3,000 lb. The material from which the beam is made is Douglas fir, structural grade. Determine the minimum size of the beam based upon the bending stress.

The maximum bending moment for the simple beam is given by

\[
M = \frac{WL}{8} = \frac{(3000)(10)}{8} = 3750 \text{ ft-lb}
\]

from case 2 in appendix A, chart B

Referring to appendix C, table 5, we find that Douglas fir, structural grade has an allowable unit stress in bending of 1800 lb / in\(^2\). Using this figure to substitute for \( f_b \) in Eq.8 and converting M from ft-lb. to in-lb.

\[
S = \frac{M}{f_b} = \frac{3750 \text{ -12}}{1800} = 25.00 \text{ in}^3
\]

After consulting appendix C, table 2, we find that a 2 X 12 \((S = 31.641 \text{ in}^3)\) will support this load.

A complete analysis of this situation would require an investigation into the shear and deflection as well as bending. Notice also that this assumes that the 2 x 12 will be sufficiently laterally supported. This means that there must be some horizontal members along the span of the beam preventing it from buckling. This will be discussed in detail later in this text.

**Lateral Support of Beams**

A beam may fail due to buckling much in the same way a column may fail due to buckling. The concept is the same. The area above the neutral surface of the beam is in compression and therefore is subject to buckling. However, because the compressive force is not uniformly distributed throughout the beam, the methods developed earlier are not sufficient.
In order for a beam to use the full value of the allowable bending stress, \( F_b = 0.66F_y \), the beam must be laterally braced. Here is where common sense and standard theatrical construction techniques will come to play due to the complex nature involved in calculating what the value of \( F_b \) will be when the spacing of lateral bracing is excessively large. To get a rough idea

\[
\frac{L}{r} \leq 40 \quad \text{(Eq. 9)}
\]

where \( L \) = the unsupported length of the beam (in.)
\( r \) = the radius of gyration of the material being used (in.)

This is simply a rough “rule of thumb” and is intended to give you some sort of ballpark figure to work with. In most cases, other design criteria will force the use of lateral support (such as joists supporting plywood in a platform).

7 - 9 Formulas for Typical Beam Loading

Appendix A gives formulas for typical beam loading situations. These formulas were derived using calculus that cannot be discussed in the limited space in this type of text. However knowledge of calculus is not necessary to make use of these formulas.
**Review and Summary**

**Types of Beams**

*Simple.* Rests on a support at each end with ends free to rotate and translate

*Cantilever.* Fixed in rotation and translation on one end only. Unsupported on the other end

*Overhanging.* One end projects out past the supports

*Continuous.* Beam with more than two supports

*Restrainted.* Both ends of the beam are fixed in translation and rotation.

**Concentrated load**

A load which acts at a single point on a beam.

**Distributed load**

A load which acts over an area of a beam.

**Vertical shear**

Forces that tend to cause adjacent material in a beam to slide past each other.

\[ V = R - \sum_{x=0}^R L_x \]

where \( R \) = the reaction to the left of point \( x \) and \( L_x \) is the sum of the loads left of point \( x \).

**Horizontal shear**

Tends to cause fibers of a beam to slide horizontally past each other.

\[ \nu = \frac{VQ}{Ib} \]

where \( V \) = the vertical shear, \( Q \) = the statical moment with respect to the neutral axis, \( I \) = the moment of inertia and \( b \) = the width of the beam.

**Shearing stress**

\[ f_v = \frac{\nu}{dt} \frac{V}{A_w} \]

where \( f_v \) = the shearing stress on the beam, \( V \) = the maximum vertical shear, \( d \) = the depth of the beam, \( t_w \) = the web thickness, and \( A_w \) = the total area of the web.

The maximum allowable shear stress is given by

\[ f_{v-max} = 0.40 f_y \]

where \( f_y \) = the allowable yield stress for the material.

**Bending moment**

\[ M_b = M_r - M_l \]

where \( M_b \) = the bending moment, \( M_r \) = the moment due to the reaction to the left of the section studied, and \( M_l \) = the moment due to the loads to the left of the section studied.

**Deflection**

\[ D = K \frac{PL^3}{EI} \]

See Appendix A, chart B for list of formulae useful in calculating the amount of deflection in a beam.

**Flexure Formula**

\[ M_{max} = f_b S \]

where \( f_b \) = the allowable unit stress in bending and \( S \) = the required section modulus for the beam to resist the load.
Exercises and Problems

7 - 3 Determining beam reactions
1. Using the beam described in figure 10, determine the reactions \( R_1 \) and \( R_2 \) using the following:
   \[ L_1 = 700 \text{ lb.} \quad L_2 = 450 \text{ lb.} \]
   \[ X_1 = 4'\cdot0'' \quad X_2 = 10'\cdot0'' \]
   \[ D = 18'\cdot0'' \]
2. Using the beam described in figure 10, determine the reactions \( R_1 \) and \( R_2 \) using the following:
   \[ L_1 = 1400 \text{ lb.} \quad L_2 = 2250 \text{ lb.} \]
   \[ X_1 = 10'\cdot9'' \quad X_2 = 15'\cdot6'' \]
   \[ D = 30'\cdot0'' \]
3. Determine the reactions \( R_1 \) and \( R_2 \) of the beam in figure 11 using the following:
   \[ W = 200 \text{ lb.} / \text{ft.} \quad D = 15'\cdot0'' \]
4. Determine the reactions \( R_1 \) and \( R_2 \) of the beam in figure 11 using the following:
   \[ W = 65 \text{ lb.} / \text{ft.} \quad D = 10'\cdot0'' \]

7 - 4 Vertical shear in beams
5. Determine the value of vertical shear in the beam from problem 1. Draw the shear diagram for this beam.
6. Determine the value of vertical shear in the beam from problem 2. Draw the shear diagram for this beam.
7. Determine the value of vertical shear in the beam from problem 3. Draw the shear diagram for this beam.
8. Determine the value of vertical shear in the beam from problem 4. Draw the shear diagram for this beam.

7 - 5 Horizontal shear in beams
9. Assuming the beam in problem 1 to be a wooden beam, calculate the value of horizontal shear.
10. Assuming the beam in problem 3 to be a wooden beam, calculate the value of horizontal shear.

7 - 6 Bending moments in beams
11. There is a simply supported beam which is 12'\cdot6'' long, with a concentrated load that is placed 5'\cdot0'' from the left side. Draw a loading diagram, a shear diagram and a bending moment diagram for this beam.
12. There is a simply supported beam which is 10'\cdot0'' long. Applied to this beam is a uniformly distributed load of 100 lb. / ft. Draw a loading diagram, a shear diagram and a bending moment diagram for this beam.
7 - 7 Deflection in beams
13. Calculate the deflection in a 10'-0" long simply supported Douglas fir 2 x 10 beam with a concentrated load of 1000 lb. placed in the center of the beam. Is this deflection too great for most common theatrical practices? Remember that there are charts useful for calculating deflection in common beams in Appendix A.
14. How much load can be placed on a simply supported, 3 x 5.7, S-shaped I-beam assuming deflection is limited to \( \frac{1}{360} \) of the beams 16'-0" length?

7 - 8 Designing a member to resist calculated shear and bending
15. You have calculated the value for horizontal shear to be 8500 lb. / in\(^2\). What type of material is necessary to resist this value of horizontal shear?
16. What is the maximum horizontal shear that a piece of no. 2 Douglas fir can withstand?
17. What is the maximum allowable bending moment a piece of 2 x 2 square steel tube can take before failing in bending? What about 2 x 2 aluminum square tube?
18. Given the beam in problem 12, determine what size wooden beam is necessary to resist bending. What size steel beam is necessary to do the same job? What about aluminum?
Chapter 8  Built Up Cross-Sections

8 - 1  Introduction to Built Up Cross-Sections

Up to now, we have only discussed homogeneous structural members, that is to say, members composed of a single, uniform material. A 2 X 4 and a steel I-beam are examples of homogeneous structural members. A problem involving a built up cross section is one in which there are two or more different materials and/or shapes fastened securely and working together. When examining a built up cross section, you cannot simply consult a table or chart to find such values as the yield stress or the moment of inertia. These values must be calculated on an individual basis.

When calculating any values based on a built up cross-section, it is important to realize that it is assumed that the individual pieces which make up the built up cross-section are homogeneous, that is to say one solid piece. Simply nailing, screwing or stapling wooden pieces together do not necessarily make the piece homogeneous. Proper use of glue and clamping make the assumption closer to the truth. Quality control and craftsmanship are very important if you expect these calculations to accurately reflect reality.

8 - 2  Centroid

The centroid is a fundamental property of the geometric shape of a structural member and has nothing to do with the material the member is composed of. The centroid is often called the center of gravity or center of mass, however these generally refer to masses while the centroid applies to lines, areas or volumes, quantities without mass. The centroid of an area is a point at which all the area may be considered to be concentrated. This is a somewhat difficult concept to visualize. Lets look at a simple cross section (such as a square) and determine the centroid. Figure 1 shows a simple square cross section. The dashed lines indicate the position of the centroid, which in this case is located at the center of the section. For most structural shapes, a similar observational process will determine the position of the centroid. If a section possesses a line of symmetry then the centroid will fall along that line of symmetry. If it possesses two lines of symmetry (as our example in figure 1 does), then the centroid will be located at the intersection of the two lines of symmetry. In a rectangle, the centroid can be found by connecting the mid point of one side to the mid point of the opposite side in both the X and Y directions. Where the two connect is the position of

Figure 1. Square cross section. Dashed lines indicate centroid position.
the centroid. In the case of a triangle, connecting the mid point of a side with the intersection of the opposite two sides places the centroid location. Where all three of these lines intersect (really only two lines are necessary), will be the position of the centroid (note that this will be 1/3 the height or length of the triangles leg). The centroid of a circle is simply the center of the circle.

It is sometimes the case that there is only one (or even no) line of symmetry. In this case we must turn to mathematics. The statical moment of an area is a purely academic concept. It is defined as the area multiplied by the distance from an axis to the centroid of that area. If an area is divided into a number of parts, the sum of the statical moments of the parts is equal to the statical moment of the entire area. Mathematically speaking

\[ Ax = \sum_{n=1}^{t} A_n x_n \]  

where \( Ax \) = the total statical moment
\( A \) = the total area of the shape in question\((in^2)\)
\( x \) = the distance from the given axis to the centroid in the X (or Y) direction \((in.)\)
\( n \) = an arbitrary counting holder. It begins at 1 and goes to \( t \) (unitless)
\( t \) = total number of areas being studied (unitless)

In most cases, it is convenient to choose the axis as either the top or bottom surface of the built up cross section being considered. The procedure for using Eq.1 begins with breaking the section under consideration up into a number of rectangles, triangles and circles. It is a simple matter to visually determine the location of the centroid of these shapes. The area for each section is computed and multiplied by the distance the centroid for that area is from the chosen axis. These statical moments are then added together to give the statical moment of the entire section as shown in Eq.1.

Sample problem 1

Locate the centroid of the shape shown in figure 2.
This composite section is made of two regular areas; a rectangle and a triangle. Applying Eq.1 and assuming the axis to be the lower surface of the section

\[ Ax = \sum_{i=1}^{n} A_i x_i \]
\[ Ax = A_1 x_1 + A_2 x_2 \]
\[ Ax = 36 \times 5 \times 27 \times \text{in}^2 \]
\[ Ax = 243 \text{ in}^3 \]

The total area is simply the sum of the individual areas of the rectangle and the triangle

\[ A = A_1 + A_2 \]
\[ A = 36 + 27 = 63 \text{ in}^2 \]

Using this result we can solve the Eq.1 to determine the distance from the lower surface of the section.

\[ Ax = 243 \]
\[ 63x = 243 \]
\[ x = 3.86 \text{ in} \]

The same procedure is followed for the Y axis, which we can take to be the left side of the section.

\[ Ay = \sum_{i=1}^{n} A_i y_i \]
\[ Ayx = A_1 y_1 + A_2 y_2 \]
\[ Ay = 36 \times 5 \times 27 \times \text{in}^2 \]
\[ Ay = 234 \text{ in}^3 \]
The total area is still the same
\[ Ay = 234 \]
\[ 63y = 234 \]
\[ y = 3.71 \text{ in} \]

The centroid is located 3.86 in. up from the bottom of the section and 3.71 in. to the right of the left side of the section.

8 - 3 Moment of Inertia

The moment of inertia related to an element of an area is defined as the product of the area of the element and the square of the distance from the axis to the element. Such an element of area would have to have an infinitesimal or differential (very small) width.

The moment of inertia of the entire area is the summation of the moments of inertia of all the elements that make up the total area.

When the width of the element of area is larger than a theoretical differential width, only an approximation of the moment of inertia can be obtained. The approximate moment of inertia may be expressed as

\[
I = \sum_{n=1}^{i} a_n y_n^2 \quad (\text{Eq.2})
\]

where
- \( I \) = the moment of inertia (in\(^4\))
- \( a \) = the area of the element (in\(^2\))
- \( y \) = the distance to the element from the chosen axis (in.)

As the width of these elements decrease, the number of elements would increase and the moment of inertia of the area would become more exact. To obtain the exact moment of inertia, the elemental area would have to be reduced to an infinitesimal small value, thus increasing the number of elements to infinity. The solution of this problem requires the use of calculus and is beyond what is necessary in the theatrical field.

Parallel axis theorem

In most cases, the moment of inertia for a specific shape will be found in a chart or table. It is only necessary to calculate a value for the moment of inertia in the case of a built up cross section. The parallel axis theorem provides a convenient relationship between the moment of inertia of an area with respect to its centroidal axis and with respect to an axis parallel to the centroidal axis.
The parallel axis theorem may be expressed as

\[ I = \sum_{i=1}^{n} I_i + A_i d_i^2 - C^2 \]  

(Henry Grillo)  

(Eq.3)

where

- \( I \) = the moment of inertia of the built up cross section (in^4)
- \( I_i \) = the moment of inertia of the individual materials making up the built up cross section (in^4)
- \( A_i \) = the cross sectional area of the individual materials making up the built up cross section (in^2)
- \( d_i \) = distance from the centroid of the individual materials to the common axis (usually the lower or upper surface of the built up cross section) (in.)
- \( C \) = the distance from the common axis to the centroidal axis of the built up cross section (in.)

**Section modulus**

The value for \( S \) in the flexure formula (Eq.8 in chapter 7) is known as the section modulus. In many cases this value will be easily found in a chart for structural shapes. However when dealing with built up cross sections this value must be calculated. The section modulus is given by

\[ S = \frac{I}{C} \]  

(Eq.4)

where

- \( S \) = section modulus (in.\(^3\))
- \( I \) = the moment of inertia (in.\(^4\))
- \( C \) = the distance to the extreme fiber from the centroidal axis (in.)

**Sample problem 2**

Find the effective moment of inertia of the 4 X 8 stress skin platform, built with 1 X 6 (3/4"x5 1/2") stringers, ¾" plywood lid and ¼" plywood bottom skin. The moment of inertia of ¾" plywood is 0.197 in.\(^4\)/ft, of ¼" plywood is 0.009 in.\(^4\)/ft and 1 X 6 10.398 in.\(^4\).

First the centroid for the built up cross section needs to be determined. To make this calculation we use Eq.1
This tells us that the centroid lies along a line parallel to the bottom surface, 4.21 in. from the bottom surface. We could repeat the above procedure to find the centroidal line in the Y axis, however it is not necessary for this problem.

Now we can use the parallel axis theorem (Eq.3) to calculate the moment of inertia for the stress skin platform.

\[
I = \sum_{i=1}^{n} \left( I_i + A_i \left[ \mu - C_i \right]^2 \right)
\]

\[
I = (0.788 + 36(6.125 - 4.21)^2) + (0.036 + 12(0.125 - 4.21)^2) + 4 \left( 0.398 + 4.125(4.21 - 4.21)^2 \right)
\]

\[
I = 132.81 + 200.28 + 65.75 = 398.84 \text{ in}^4
\]

Notice that the moment of inertia (I) for the \(\frac{3}{4}\)" and \(\frac{1}{4}\)" plywood is given as a value per foot of plywood. In both cases there is four feet, hence a value of 0.788 for the \(\frac{3}{4}\)" plywood and 0.036 for the \(\frac{1}{4}\)" plywood.
Sample Problem 3

Given the stress skin platform from sample problem 2 what load will the platform support? What will be the deflection?

To solve this problem we need to treat the platform as if it were a simply supported beam. This is not entirely correct, however the approximation will produce conservative results.

To begin with we need to calculate the maximum bending moment for the built up cross section. This is given by the flexure formula (chapter 7, Eq.8).

\[ M_{\text{max}} = f_b S \]

For \( f_b \), we will use the value for the material which is the farthest from the centroidal axis, in this case the \( \frac{1}{4}'' \) plywood (\( f_b = 1400 \frac{\text{lb}}{\text{in}^2} \)). We need to calculate the section modulus using Eq.4.

\[ S = \frac{I}{C} \]

\[ = \frac{398,84}{4.21} \]

\[ = 94.74 \text{ in}^3 \]

Now we can make use of the flexure formula

\[ M_{\text{max}} = f_b S \]

\[ M_{\text{max}} = 1400 \cdot 94.74 \]

\[ M_{\text{max}} = 132630.9 \text{ in} \cdot \text{lb} \]

\[ = 11052.6 \text{ ft} \cdot \text{lb} \]

This tells us that the maximum bending moment this beam (platform) can withstand is 11052.6 ft-lb. What uniformly distributed load does this correspond to? Consulting chart B, appendix A we find that the bending moment for a simply supported beam with a uniformly distributed load is given by

\[ M = \frac{WL}{8} \]

Using the maximum bending moment we calculated and the length of the beam, we can calculate \( W \), the value of the uniformly distributed load.

\[ M = \frac{WL}{8} \]

\[ 11052.6 = \frac{W(8 \text{ ft})}{8} \]

\[ W = 11052.6 \text{ lb} \]

or 1381.6 \( \frac{\text{lb}}{\text{ft}} \). Because the platform is 4 ft wide and 8 ft long, this value divided by 32 (the total area for the platform) will give us a value of 345.4 \( \frac{\text{lb}}{\text{ft}^2} \), a very high value for a platform with 1 x 6 framing!

To calculate the amount of deflection this platform will experience while under its maximum load, we consult appendix A, chart B and find the equation for deflection is
\[ D = \frac{5WL}{384EI} \]

In our example, \( W = 11052.6 \text{ lb.} \), \( L = 96 \text{ in.} \), \( E_{\text{(Douglas fir)}} = 1,500,000 \text{ lb/}\text{in}^2 \), and \( I = 398.84 \text{ in}^4 \). Using these values in the equation for deflection we see that

\[ D = \frac{5(11052.6)(96^3)}{384(1500000)(398.84)} \]

\[ D = 0.21 \text{ in} \]

which is less than 1/360 of the total span.

### 8 - 4 Built up Wooden Columns

It is fairly common to see built up columns as well as beams. Some common built up columns are the “hog’s trough” or “L-section" legs, square section legs or “I-beam” legs.

You will remember from section 5 - 2, that to design compressive columns there are two things of import, the radius of gyration and the allowable unit stress in compression, \( F_c \).

The radius of gyration is given by

\[ r = \sqrt{\frac{I}{A}} \]  
(Simplified Engineering for Architects and Builders p.167)  \hspace{1cm} (Eq.4)

where

- \( r \) = the radius of gyration (in.)
- \( I \) = the minimum moment of inertia of the built up column (as determined earlier this chapter) (in\(^4\))
- \( A \) = the total cross sectional area of the built up column (in\(^2\))

Because the pieces of the built up column will almost always be rectangular shapes, we can simplify Eq.2 for a rectangle. The simplified equation is given by

\[ I = \frac{bd^3}{12} \]  
(Simplified Mechanics and Strength of Materials)  \hspace{1cm} (Eq.5)

where

- \( I \) = the moment of inertia for a rectangular cross section (in\(^4\))
- \( b \) = the length of the side parallel to the axis being studied (in.)
- \( d \) = the length of the side perpendicular to the axis being studied (in.)
The allowable unit stress in compression is given by

\[ f_c = \frac{3.619E}{l^2/r} \]  \hspace{1cm} (Eq.6)

where \( f_c \) = the allowable unit stress in compression (\( \frac{\text{lbs}}{\text{in}^2} \))

\( E \) = the modulus of elasticity of the material used (the minimum if there are different species involved) (\( \frac{\text{lbs}}{\text{in}^2} \))

\( l \) = the unsupported column length (in.)

\( r \) = the radius of gyration for the built up section (in.), as determined using Eq.4.

Once these two values are known, the process is exactly the same as was discussed in section 5 - 2.

Sample problem 3
The built up column in Figure 5 is made from ¾” thick Douglas fir and is 6 feet long. What is the maximum load this column can carry?

We first need to determine the moment of inertia for this section. Because it is a symmetric section, it does not matter whether we use the moment of inertia in the X-X axis or the Y-Y axis. To do this we need to calculate the centroid. Let's choose the bottom surface (as drawn) to be the reference axis.

\[ Ax = \sum_{x=1}^{3} A_x \cdot x \]

\[ Ax = 3(3.625) = 2.44(1.625) \]

\[ Ax = 14.84 \]

The total area is given by

\[ A = A_1 + A_2 = 3 + 2.44 = 5.44 \]

\[ 5.44x = 14.84 \]

\[ x = 2.72 \text{ in.} \]
Now using Eq.5

\[ I = \frac{bd^3}{12} \]

. \( I_1 = \frac{(0.75)(4)^3}{12} = 4.0 \)

. \( I_2 = \frac{(4)(0.75^3)}{12} = 0.141 \)

These two values will be needed in the calculation for the moment of inertia of the entire section as given in Eq.3

\[ I = \sum_i \left[ \frac{i}{A} (i - C)^2 \right] \]

\[ I = \frac{0 + 3(3.625 - 2.73)}{141 + 2.44(1.625 - 2.73)} \]

\[ I = 9.52 \text{ in}^4 \]

Now using Eq.4 to find the radius of gyration

\[ r = \sqrt{\frac{I}{A}} \]

\[ r = \sqrt{\frac{9.52}{5.44}} = 1.32 \text{ in.} \]

This is the first major piece of information necessary to solve the problem. The second is the allowable unit stress in compression for the material, which is given by Eq.6

\[ f_c = \frac{3.619E}{\sqrt[3]{r}} \]

\[ f_c = \frac{3.619(1800000)}{\sqrt[3]{1.32}} \]

\[ f_c = 2189.5 \text{ lb.} \]

applying the direct stress formula we get

\[ P = f_c A \]

\[ P = (2189.5)(5.44) \]

\[ P = 11911 \text{ lb.} \]
**Review and Summary**

**Centroid**
The center of mass of a cross section. An imaginary point given by

\[ Ax = \sum_{n=1}^{N} A_n x_n \]

where \( A_n \) = area of each section being studied and \( x_n \) = the distance to the center of the area from a reference axis.

**Moment of Inertia**
Moment of inertia is defined as the product of the sum of the products of a differential area and the square of the distance from an axis.

\[ I = \sum_{n=1}^{i} a_n y_n^2 \]

where \( a_n \) = area of one element, and \( y_n \) = the distance to that element from an axis.

**Parallel Axis Theorem**
A relationship between the moment of inertia of an area with respect to its centroidal axis and with respect to an axis parallel to the centroidal axis.

\[ I = \sum_{i=1}^{n} \left( I_i + A_i \left( d_i - C \right)^2 \right) \]

where \( I \) = the moment of inertia of the built up cross section, \( I_i \) = the moment of inertia of the individual materials, \( A_i \) = the cross sectional area of the individual materials, \( d_i \) = the distance form the centroid of the individual materials to the common axis and \( C \) = the distance from the common axis to the centroidal axis of the built up cross section.

**Radius of gyration of a built up Cross-section**

\[ r = \sqrt{\frac{I}{A}} \]

where \( I \) = the minimum moment of inertia of the built up column and \( A \) = the total cross sectional area of the built up column.
(a) Exercises and Problems

8 - 2 Centroid
Calculate the location of the centroid of the following shapes

1. \( 4'' \)
   \[ \begin{array} {c}
   3/8'' \text{ thick} \\
   \frac{1}{4}'' \text{ thick} \\
   3''
   \end{array} \]

2. \( 4'-0'' \)
   \[ \begin{array} {c}
   3/4'' \text{ ply} \\
   4 1/2'' \\
   \frac{1}{4}'' \text{ ply}
   \end{array} \]

8 - 3 Moment of inertia
3. Calculate the moment of inertia for the built up cross section for the beam given in problem 1
4. Calculate the moment of inertia for the built up cross section for the beam given in problem 2

8 - 4 Built up wooden columns
5. A platform leg is constructed using two pieces of 1 x 4 glued and stapled edge to face to form an “L-section” leg. Calculate the moment of inertia and the radius of gyration for this leg. If it is 2’-0” long, how much load can safely be placed on it assuming it is fixed in translation and rotation on both ends?
6. A 3/4” plywood box is built to serve as a platform leg. It measures 4” x 4” x 1’-6” long. Assuming it is fixed in rotation and free to translate on the top and fixed in translation but free to rotate on the bottom, what is the maximum load applicable to this leg?
Chapter 9 Mechanical Wood Fasteners

Although there are certainly mechanical fasteners for metal members as well as wooden members, this chapter will limit its discussion to mechanical fasteners used in wooden members. The reason for this is two fold; first, in many cases, mechanical fasteners used in to connect wood to steel are not used in a weight-bearing situation. Tec screws and pop rivets connecting plywood to steel flats or platforms are not really under great strain. This is not to say that careful though should not be paid in the choice of fastening device, but it is more of a question of stagecraft than mechanical engineering. The second reason for not discussing mechanical fasteners used in metal connections is that when the fastener is being used in a weight bearing situation, it is usually either seeing a shear force such as two steel plates bolted together or it is seeing a tensional force such as a steel flat bolted under a second steel flat and the entire unit supported from above. In either of these cases, a reasonable approximation can be made using discussions found earlier in this text.

Accurately predicting the behavior of a wooden system has to be taken with a grain of salt. Wood is a natural product and is graded by the human eye. Attempts have been made to make this grading process as accurate as possible, but it is still up to one person whether a piece of lumber is no.1 or no. 2. The performance characteristics will also greatly depend upon the condition of the wood. Considerations such as how dry is the lumber, will it stay that dry under working conditions, has the lumber been abused, is it split, etc.…must all be taken into account. These problems become even more exaggerated when we introduce mechanical fasteners to the system. However, this is unavoidable. You must remember that in many cases, the numbers calculated are approximations or averages.

There are three different types of mechanical fasteners used in wooden construction. They are nails (including wire staples), bolts and screws.

9 - 1 Nails and nail joints

Nails are probably the most common form of mechanical fastener. There are many types, sizes and surface treatments of nails. In general nails give stronger joints when driven into the side grain of wood rather than the end grain. Nails also perform better when subjected to lateral loading rather than withdrawal loading, so a nailed joint should always be designed to resist lateral forces whenever possible.

Figure 1. (a) Lateral loading (b) Withdrawal loading

Nails and nail joints
Withdrawal resistance

The resistance of a nail to pull out is a function of the specific gravity of the wood, the diameter of the nail and the depth of penetration of the nail. For common nails driven into the side grain of seasoned wood that will remain seasoned the allowable withdrawal resistance is given by

\[ P = 1380G^{2.5}D \]  
(\text{Wood Technology in the Design of Structures p.136})  
(\text{Eq.1})

where \( P \) = the allowable load in pounds per inch of penetration per nail
\( G \) = the specific gravity of the species of wood used
\( D \) = the nail diameter (in.)

The specific gravity of various species of wood is given in appendix C, table 1. These allowable loads are 1/5 of the average ultimate load. This should take care of any variation of the specific gravity or variations of the surface characteristics of the nail used. The withdrawal resistance of nails driven into the end grain is only half of the value given by Eq.1.

Lateral load resistance

The allowable load for common nails driven into the side grain of seasoned wood can be determined from the following

\[ P = KD^{1.5} \]  
(\text{Wood Technology in the Design of Structures p.139})  
(\text{Eq.2})

where \( P \) = the allowable lateral load per nail at rated penetration (lbs.)
\( K \) = a constant related to the density of the wood (see table 9 - 1)
\( D \) = the diameter of the nail (in.)

In the case for lateral load resistance, instead of using the specific gravity for each species, we use the constant \( K \) as given in table 9 - 1. Here the different species are broken up into common groups (given in appendix C table 1). Allowable lateral load drops off rapidly with less than the rated penetration and should be avoided.

<table>
<thead>
<tr>
<th>Group</th>
<th>Rated Penetration</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group I</td>
<td>10 diameters</td>
<td>2040</td>
</tr>
<tr>
<td>Group II</td>
<td>11 diameters</td>
<td>1650</td>
</tr>
<tr>
<td>Group III</td>
<td>13 diameters</td>
<td>1350</td>
</tr>
<tr>
<td>Group IV</td>
<td>14 diameters</td>
<td>1080</td>
</tr>
</tbody>
</table>

Table 9 - 1
K for various species of wood

Note: Species grouping are given in appendix C table 1 (\text{Wood Technology in the Design of Structures p.140})

9 - 2 Bolts and Bolted Joints

Bolts are generally used to resist loads perpendicular to the axis of the bolt, that is to say we usually laterally load bolts or apply shear forces to bolts. Although we do sometimes load bolts so as to resist tension (such as hanging a piece of scenery from an eye-bolt), this is more of a function of the compressive bearing area under washers and compressive strength of the wood.
The shear load capacity of a bolt depends upon the ratio of the length, \( l \), of the bolt in the main member and its diameter \( d \), and on the species of wood. The main member, of a three-member system, is generally regarded as the middle of the three members. The two outer members are known as the side plates (see figure 2).

The tabulated values in Appendix C table 14 are for three member joints whose side members are at least half as thick as the main member. For thinner side members, \( l \) is taken to be twice the thickness of the thinner member.

For a two-member joint, the allowable load is half the allowable load for a value of \( l \) equal to twice the thickness of the thinner member. Therefore, if you had a piece of 1 X 4 bolted to a piece of 2 X 4, \( l \) would be equal to 1.5 (0.75 X 2).

The integrity of a bolted joint can be seriously hurt if the bolts are spaced too close. The minimum bolt spacing in a row parallel to the load axis should be four bolt diameters for both parallel-to-grain and perpendicular-to-grain loading (Wood Technology in the Design of Structures p.153).

You will notice that the values in appendix C, table 14 only covers bolts as small as \( \frac{1}{2} \)”. To calculate values for smaller bolts, simply multiply the value for a \( \frac{1}{2} \)” bolt by 0.75 to get the value for a \( \frac{3}{8} \)” bolt. To find the value for a \( \frac{5}{16} \)” bolt, multiply the value for a \( \frac{1}{2} \)” bolt by 0.63, and for a \( \frac{1}{4} \)” bolt, multiply the value for a \( \frac{1}{2} \)” bolt by 0.50. These values are all for the design value parallel to the grain. To find the design value perpendicular to the grain, multiply these values by 0.50 (Wood Technology in the Design of Structures p.153).

**9 - 3 Wood Screws and Lag Bolts**

Screws are in some ways superior to nails. Although they are not nearly as easy or as quick to use as a nail is (especially a pneumatic driven nail), they do provide a much more positive attachment than a nail does. Screws can be tightened down to squeeze two members together (which helps produce a stronger glue joint). For dynamic loading situations, such as a piece that is moved frequently, a screw is much less likely to work its way out than a nail is.

Although it is generally not recommended that nails be relied on for computed loading in withdrawal, screws are not so limited and are often chosen where the details of the connection require such a loading. A no.9
wood screw has the same diameter as a 20d nail (0.177 in.). The withdrawal resistance in dry Douglas fir is 131 and 49 lb. per inch of penetration, respectively. A ¼” lag bolt and a 50d nail are about the same diameter (0.25 in. vs. 0.244 in.) and the withdrawal resistance are 232 and 63 lb. per inch of penetration respectively.

**Wood Screws**

Pilot holes for wood screws are very important in order to have the expected withdrawal resistance. The pilot hole diameter in softwoods should be about 70% of the diameter of the screw. In hardwoods, the pilot hole should be about 90% of the screw diameter (Wood Technology in the Design of Structures p.211).

Spacing, end and edge rules for screw placement are not well established. Screws should be placed so as to prevent wood splitting. For screws into the end grain of a member, the values given in appendix C table 13 should be halved. Also, it should be realized that typical wood screws are threaded up to 2/3 of their length. When calculating the withdrawal resistance, the length of penetration is the *threaded* length of penetration.

Appendix C, table 13 gives design values for lateral resistance for wood screws. It should be pointed out here that the following discussion is for *wood screws*, not drywall screws. Drywall screws are not meant to be loaded laterally and they have very little strength in this regard and their use in lateral shear should be avoided. These values are for single shear, seasoned wood that is the most common situation you are likely to encounter in the theatre world. These values apply for any direction of lateral load to the grain, but assume that the screw is into the side grain, not the end grain of the wood. For the values of a screw into the end grain, use 2/3 of the given value. In figuring allowable lateral loads, penetration includes shank penetration and is not limited to threaded length as is the case for withdrawal load resistance.

**Lag Bolts**

Withdrawal resistance of a lag bolt is a function of its diameter, the length of the threaded part in engagement with the member containing the point, and the specific gravity of the wood. See appendix C, table 11 for these tabulated values. Note that these values are given in pounds per inch of penetration of the threaded part into the member holding the point and they assume axial load perpendicular to the side grain. When the lag bolt is in the end grain of the lumber, these values must be multiplied by 0.75 (Wood Technology in the Design of Structures p.207).

Lateral resistance is given in appendix C table 12. These values assume single shear in seasoned wood and the lag bolt put into the side grain of the wood. For lag bolts in the end grain, use half of the given value.

Lag bolts should be used with pilot holes. The size of the pilot hole should be 75% of the shank size of the lag bolt and washers should always be used under the heads of the bolt.

**Adhesives**

The subject of adhesives is a very long and complicated one. As a general rule of thumb, any good aliphatic resin or polyvinyl acetate (wood glue and white glue), when used correctly, will give you a joint that will be stronger than the wood joined. The key here is “*when used correctly*”. Rarely in scenic construction are these glues used correctly. Gluing two pieces of 1 x 4 end to face and shooting a few staples into it doesn’t qualify as clamping in most cases. This is not to say glue shouldn’t be used or you should put clamps on every piece of scenery built until the glue properly sets, however it does mean that the structural nature of the glue joint cannot be mathematically
determined. There are simply too many factors that cannot be predicted. My own opinion is that if you have to rely upon a glue joint, you should probably reconsider your construction process.
Review and Summary

Withdrawal resistance in nails

\[ P = 1380G^{2.5}D \]
where G = the specific gravity of the species of wood and D = the nail diameter.

Lateral resistance in nails

\[ P = KD^{1.5} \]
where K = a constant given in table 9 - 1 and D = the diameter of the nail.

Bolt spacing

Bolts should never be placed closer than 4 diameters apart from each other.
Exercises and Problems

9 - 1 Nails and nail joints
1. Calculate the amount of force required to withdraw a 16d common nail driven 2” into the end grain of a piece of Douglas fir. How does this force differ if it were driven the same distance into a piece of white oak?
2. Calculate the withdrawal force required to pull an 8d common nail, driven in 1 ½”, from the side grain of a piece of white pine.
3. What is the lateral resistance of a 10d common nail when driven to its rated penetration in the end grain of a piece of Douglas fir? What about the side grain? What about maple?

9 - 2 Bolts and bolt joints
4. Two pieces of Douglas fir 2 x 8 are bolted together using ½” bolts spaced 6” apart. What is the allowable load parallel to the grain?
5. Two pieces of white pine 1 x 6 are bolted together using ¼” bolts spaced 3” apart. What is the allowable load parallel and perpendicular to the grain?
6. What is the minimum bolt spacing for 3/8” bolts? ¼” bolts?

9 - 3 Wood screws and lag bolts
7. What is the withdrawal resistance for a no. 10 wood screw when used in the end grain of white pine and is driven to 1” depth?
8. What is the withdrawal and lateral resistance for a ¼” lag bolt driven 2” into the side grain of oak?
Conclusion

The process of preparing a text such as this was a great challenge. There is a vast amount of information available on the subject and much of it goes into more detail than is usually necessary for a theatrical technician. All but a few of my source texts were books written for architects or beginning to intermediate engineering students. What I attempted to do was distill down the complex mathematics and theory into a form more useful to students of scenic construction.

My goal was not to create a stand alone text, but to write a useful teaching aid to help the student by giving them an auxiliary source of information. This text is meant to be used in a classroom setting to give the student a place to go for additional information or to reiterate information covered in class.

My process began in the summer of 1995. I started with compiling a list of everything I felt should be in a text of this sort. It is the result of that list that you see before you. There were some additional subjects which were on that list that, upon researching, I realized were far beyond the scope of what I could conceivably cover in the time and space allotted.

As I began writing, it became apparent that some material which I had at first considered simple and only need to be touched upon actually needed much more in depth coverage. At the same time, subjects which I thought would need careful and explicit discussion turned out to be made simpler and easily understandable. In the end, I feel that I have put together a complete covering of the subject of structural engineering as it relates to the field of scenic construction.

Scenic construction has many standards taught in schools across the country. Most of these conventions have developed over years of trial and error and have led, in most cases, to good, solid scenery. Lately there seems to be a trend in theatre to be designing scenery that goes away from what used to be typical. Platforms and flats are being asked to do things they rarely were asked to do years ago and technical directors and theatrical technicians are finding that they have to improvise and alter many of the standard techniques they have in their repertoire.

The methods discussed in the preceding pages are intended to give the theatrical technician tools to make use of in order to confidently engineer scenic units that might need to support loads or span distances which have not been tried in the past. In addition, it gives the theatrical technician the skills and vocabulary necessary to communicate with a professional engineer in the case that such a person should be called upon. It is conceivable that an engineer might simply point you in the correct direction to allow you, with the skills learned in this text, to solve the problem yourself. There will always be cases when a professional engineer will need to be consulted on a scenic unit, but it always helps to be able to talk in an educated, intelligent manner when consulting one.

Please remember that mastery of this material does in no way make you an engineer. Engineers go to school for many years to learn their trade. Although accurate, the information given in this text is simplified and truncated so as to avoid the use of complex mathematics. When lives are at stake, it cannot hurt to ask the help of a professional.
APPENDIX A

Formulas and constants for beams and columns

Properties of sections
# TABLE 1
**GENERAL DESIGN VALUES FOR COMMON MATERIALS**

1) **Yield stress:**

<table>
<thead>
<tr>
<th>Material</th>
<th>Value (lb/in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel (A36)</td>
<td>36,000</td>
</tr>
<tr>
<td>Aluminum (6061-T6)</td>
<td>21,000</td>
</tr>
<tr>
<td>Douglas Fir, S. Yellow Pine</td>
<td>1600</td>
</tr>
<tr>
<td>Hemlock, Ponderosa Pine, Spruce</td>
<td>680</td>
</tr>
<tr>
<td>Redwood</td>
<td>1250</td>
</tr>
<tr>
<td>Northern White Pine</td>
<td>950</td>
</tr>
<tr>
<td>Plywood (all interior grades)</td>
<td>2100</td>
</tr>
</tbody>
</table>

2) **Allowable unit stress in bending, f_b (generally, 0.66 times the materials yield stress):**

<table>
<thead>
<tr>
<th>Material</th>
<th>Value (lb/in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel (A36)</td>
<td>24,000</td>
</tr>
<tr>
<td>Aluminum (6061-T6)</td>
<td>14,000</td>
</tr>
<tr>
<td>Douglas Fir, S. Yellow Pine</td>
<td>1050</td>
</tr>
<tr>
<td>Hemlock, Ponderosa Pine, Spruce</td>
<td>450</td>
</tr>
<tr>
<td>Redwood</td>
<td>825</td>
</tr>
<tr>
<td>Northern White Pine</td>
<td>625</td>
</tr>
<tr>
<td>Plywood (all interior grades)</td>
<td>1400</td>
</tr>
</tbody>
</table>

3) **Allowable unit stress in shear, Fv (generally, .4 times the materials yield stress):**

<table>
<thead>
<tr>
<th>Material</th>
<th>Value (lb/in²)</th>
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</thead>
<tbody>
<tr>
<td>Steel (A36)</td>
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<tr>
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<td>8,400</td>
</tr>
<tr>
<td>Douglas Fir, S. Yellow Pine</td>
<td>90</td>
</tr>
<tr>
<td>Hemlock, Ponderosa Pine, Spruce</td>
<td>85</td>
</tr>
<tr>
<td>Redwood</td>
<td>80</td>
</tr>
<tr>
<td>Northern White Pine</td>
<td>60</td>
</tr>
<tr>
<td>Plywood (all interior grades)</td>
<td>175</td>
</tr>
</tbody>
</table>

3) **Modulus of Elasticity**

<table>
<thead>
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<th>Material</th>
<th>Value (lb/in²)</th>
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</thead>
<tbody>
<tr>
<td>Steel (A36)</td>
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<tr>
<td>Aluminum (6061-T6)</td>
<td>10,000,000</td>
</tr>
<tr>
<td>Douglas Fir</td>
<td>1,500,000 to 1,900,000</td>
</tr>
<tr>
<td>Ponderosa, White Pine, Spruce</td>
<td>990,000 to 1,240,000</td>
</tr>
<tr>
<td>Redwood</td>
<td>960,000 to 1,340,000</td>
</tr>
</tbody>
</table>
CHART A
EFFECT OF END CONDITIONS IN COMPRESSION

<table>
<thead>
<tr>
<th>Buckled shape of column is shown by dashed lines</th>
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</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram A" /></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Recommended K value</th>
<th>0.65</th>
<th>0.80</th>
<th>1.2</th>
<th>1.0</th>
<th>2.1</th>
<th>2.0</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>End condition code</th>
<th><img src="image7.png" alt="Code A" /></th>
<th>Rotation fixed and translation fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image8.png" alt="Code B" /></td>
<td>Rotation free and translation fixed</td>
<td></td>
</tr>
<tr>
<td><img src="image9.png" alt="Code C" /></td>
<td>Rotation fixed and translation free</td>
<td></td>
</tr>
<tr>
<td><img src="image10.png" alt="Code D" /></td>
<td>Rotation free and translation free</td>
<td></td>
</tr>
</tbody>
</table>

*Source: Manual of Steel Construction 8th edition*
Values for Typical Beam Loading

Chart A shows values for typical beam loading situations. Formulas given are maximum bending moment (M), maximum vertical shear (V) and maximum deflection (D). In addition, for cases 1, 3, 4, 5 and 6 give a formula for the equivalent tabular load (ETL). The equivalent tabular load is a hypothetical uniformly distributed load which when applied to the beam will produce the same magnitude of maximum bending moment.

\[ M = \text{maximum bending moment. Units will be in foot} \times \text{pounds if the load is in pounds and the length is in feet. Units will be in inch} \times \text{pounds if the load is in pounds and the length is in inches} \]

\[ V = \text{maximum vertical shear. Units will be the same as the units for the load, usually in pounds.} \]

\[ D = \text{Maximum deflection. Units will be in inches.} \]

\[ P = \text{Value for a concentrated load. Units will usually be in pounds or kips (thousand pounds).} \]

\[ W = \text{Value for a uniformly distributed load. Units will usually be in pounds per foot.} \]

\[ L = \text{Length of beam. Units will usually be in feet, however when used in the deflection formula, the units must be in inches.} \]

\[ E = \text{Modulus of Elasticity for the material the beam is made from. Units are in pounds per square inch.} \]

\[ I = \text{Moment of Inertia for the shape of the cross section of the beam. Units are in inches to the fourth power.} \]
CHART B
VALUES FOR TYPICAL BEAM LOADING

Source: Manual of Steel Construction 8th edition
Chart C
Properties of Various Geometrical Shapes

Solid Square
- Area ($A$): $d^2$
- X coordinate ($x$): $d/2$
- Moment of Inertia ($I$): $d^4/12$
- Section Modulus ($S$): $d^3/6$
- Radius of gyration ($r$): $d/\sqrt{12}$

Solid Rectangle
- Area ($A$): $bd$
- X coordinate ($x$): $d/2$
- Moment of Inertia ($I$): $bd^3/12$
- Section Modulus ($S$): $bd^2/6$
- Radius of gyration ($r$): $d/\sqrt{12}$

Hollow Square
- Area ($A$): $d_1^2 - d_2^2$
- X coordinate ($x$): $d_1/2$
- Moment of Inertia ($I$): $(d_1^4 - d_2^4)/12$
- Section Modulus ($S$): $d_1^4 - d_1^4/6d_1$
- Radius of gyration ($r$): $\sqrt{d_1^4 + d_2^4}/12$
APPENDIX B

DESIGN VALUES FOR STEEL AND ALUMINUM STRUCTURAL MEMBERS
Steel

Steel is becoming more and more prevalent in the theatre world. If is the strongest commonly available construction material per weight, and it is also the stiffest (highest value for modulus of elasticity). The process of manufacturing steel is so closely regulated that you can be quite sure that there is little, if any, variation in its structural properties (this certainly cannot be said of lumber).

Most steel used in theatrical applications is classified by the American Iron and Steel Institute as AISI C-1020 or A-36 for structural steel, or AISI M-1020 for merchant quality steel. A-36 steel (referring to its yield stress of 36,000 psi) is very uniform and free of defects and is usually used where failure would be catastrophic. A-36 steel is usually formed into large steel shapes such as I-beams (S and W), channels, large angle iron, large rectangular and square box tube, etc. Merchant quality steel is not made to the same exacting standards that A-36 steel is made and is usually formed into small angle iron, bar sized (small) channel. Merchant quality steel is useful for general scenic use in such stiffening flats or where failure would not be catastrophic.

More details on steel and its design properties can be found in the Steel Construction Manual or from literature supplied by steel manufactures.

Aluminum is much softer and much lighter than steel. It is not as strong as steel, however its strength is quite great. It can be bolted and welded using similar methods used in steel and is available in many of the same shapes.

Aluminum comes in an incredibly wide array of alloys and tempers, all of which have greatly differing strengths. One of the most common alloy is 6061 T-6, and this is the alloy discussed in this text.

S - Shape: Less common than the W - Shape in our applications. Doesn’t resist forces applied perpendicular to the web as well as the W - Shape.

W - Shape: Most common in theatrical applications. Has more resistance to lateral forces, therefore W-Shapes make good columns or long span beams with little lateral support.

C - Shape: Channel. Typically used as beams or columns. Produced in A-36 down to 3”. Smaller channel is made in merchant quality steel.

Angle: Typically used as a tab or bracket. Sometimes used as beams or columns. Made in A-36 and merchant quality down to 1/8” X 1/2” X 1/2”
### Table 1

**Allowable Unit Stresses for Columns of A36 Steel (in KSI)**

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<th>Kl/r</th>
<th>F_a (ksi)</th>
<th>Kl/r</th>
<th>F_a (ksi)</th>
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Source: Simplified Truss Design
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Source: Alcoa Structural Handbook
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<th>Weight per foot (Alum.) lb.</th>
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### TABLE 5

**DESIGN VALUES FOR S SHAPE I - BEAMS**

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Source: [Manual of Steel Construction, 8th edition](#).
### TABLE 6
DESIGN VALUES FOR AMERICAN STANDARD CHANNELS

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<td>0.412</td>
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<td>2.375</td>
<td>1.939</td>
<td>0.218</td>
<td>5.02</td>
<td>1.48</td>
<td>0.868</td>
<td>0.731</td>
<td>0.766</td>
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<td>2¼</td>
<td>2.875</td>
<td>2.323</td>
<td>0.276</td>
<td>7.66</td>
<td>2.25</td>
<td>1.92</td>
<td>1.34</td>
<td>0.924</td>
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<td>3</td>
<td>3.500</td>
<td>2.900</td>
<td>0.300</td>
<td>10.25</td>
<td>3.02</td>
<td>3.89</td>
<td>2.23</td>
<td>1.14</td>
</tr>
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<td>3¼</td>
<td>4.000</td>
<td>3.364</td>
<td>0.318</td>
<td>12.50</td>
<td>3.68</td>
<td>6.28</td>
<td>3.14</td>
<td>1.31</td>
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<td>3.826</td>
<td>0.337</td>
<td>14.98</td>
<td>4.41</td>
<td>9.61</td>
<td>4.27</td>
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<td>5.563</td>
<td>4.813</td>
<td>0.375</td>
<td>20.78</td>
<td>6.11</td>
<td>20.7</td>
<td>7.43</td>
<td>1.84</td>
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<td>6</td>
<td>6.625</td>
<td>5.761</td>
<td>0.432</td>
<td>28.57</td>
<td>8.40</td>
<td>40.5</td>
<td>12.2</td>
<td>2.19</td>
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<tr>
<td>8</td>
<td>8.625</td>
<td>7.625</td>
<td>0.500</td>
<td>43.39</td>
<td>12.8</td>
<td>106</td>
<td>24.5</td>
<td>2.88</td>
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## TABLE 9

**STRUCTURAL VALUES FOR BLACK IRON PIPE (Con’t)**

<table>
<thead>
<tr>
<th>Nominal Diameter</th>
<th>Outside Diameter</th>
<th>Inside Diameter</th>
<th>Wall Thickness</th>
<th>Weight / foot</th>
<th>Area $^2$</th>
<th>Area $^4$</th>
<th>Area $^3$</th>
<th>Area $^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 in</td>
<td>2.375 in</td>
<td>1.503 in</td>
<td>0.436 in</td>
<td>9.03 lb.</td>
<td>2.66</td>
<td>1.31</td>
<td>1.10</td>
<td>0.703</td>
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<tr>
<td>2 ½ in</td>
<td>2.875 in</td>
<td>1.771 in</td>
<td>0.552 in</td>
<td>13.69 lb.</td>
<td>4.03</td>
<td>2.87</td>
<td>2.00</td>
<td>0.844</td>
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<tr>
<td>3 in</td>
<td>3.500 in</td>
<td>2.300 in</td>
<td>0.600 in</td>
<td>18.58 lb.</td>
<td>5.47</td>
<td>5.99</td>
<td>3.42</td>
<td>1.05</td>
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<tr>
<td>4 in</td>
<td>4.500 in</td>
<td>3.152 in</td>
<td>0.674 in</td>
<td>27.54 lb.</td>
<td>8.10</td>
<td>15.3</td>
<td>6.79</td>
<td>1.37</td>
</tr>
<tr>
<td>5 in</td>
<td>5.563 in</td>
<td>4.063 in</td>
<td>0.750 in</td>
<td>38.55 lb.</td>
<td>11.3</td>
<td>33.6</td>
<td>12.1</td>
<td>1.72</td>
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<tr>
<td>6 in</td>
<td>6.625 in</td>
<td>4.897 in</td>
<td>0.864 in</td>
<td>53.16 lb.</td>
<td>15.6</td>
<td>66.3</td>
<td>20.0</td>
<td>2.06</td>
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<tr>
<td>8 in</td>
<td>8.625 in</td>
<td>6.875 in</td>
<td>0.875 in</td>
<td>72.42 lb.</td>
<td>21.3</td>
<td>162</td>
<td>37.6</td>
<td>2.76</td>
</tr>
</tbody>
</table>

**Source:** Manual of Steel Construction 8th edition.
| Source: Alcoa Structural Handbook |
TABLE 11

STRUCTURAL VALUES FOR ALUMINUM PIPE

Source: Alcoa Structural Handbook
TABLE 12
STRUCTURAL VALUES FOR ALUMINUM ANGLES

Source: Alcoa Structural Handbook
| Source: Alcoa Structural Handbook |

### TABLE 13

**STRUCTURAL VALUES FOR ALUMINUM I - BEAM** *(American Standard)*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Description</td>
<td>Details</td>
</tr>
</tbody>
</table>

...
TABLE 14
STRUCTURAL VALUES FOR ALUMINUM CHANNEL (American Standard)

Source: Alcoa Structural Handbook
APPENDIX C

DESIGN VALUES FOR WOODEN STRUCTURAL MEMBERS
TABLE 1

GROUPING OF SPECIES AND SPECIFIC GRAVITATES

Source: Wood Technology in the Design of Structures
### TABLE 2
SECTION PROPERTIES FOR SAWN LUMBER AND TIMBER

*Source: Wood Technology in the Design of Structures*
| TABLE 3 |
| DESIGN VALUES FOR LIGHT FRAMING AND STUDS |
| 2” to 4” Thick, 2” to 4” Wide |

*Source: Wood Technology in the Design of Structures*
TABLE 4
DESIGN VALUES FOR STRUCTURAL LIGHT FRAMING AND APPEARANCE

2” to 4” Thick, 2” to 4” Wide

Source: Wood Technology in the Design of Structures
TABLE 5

DESIGN VALUES FOR STRUCTURAL JOISTS AND PLANKS

2” TO 4” Thick, 6” and Wider

*Source: Wood Technology in the Design of Structures*
TABLE 6
DESIGN VALUES FOR POSTS AND TIMBERS
5” x 5” and Wider

*Source: Wood Technology in the Design of Structures*
TABLE 7
DESIGN VALUES FOR PLYWOOD
TABLE 8
FULL LOAD PENETRATION FOR NAILS AND SPIKES

Source: *Wood Technology in the Design of Structures*
TABLE 9
ALLOWABLE WITHDRAWAL LOADS FOR NAILS AND SPIKES

Source: Wood Technology in the Design of Structures
| Source: Wood Technology in the Design of Structures |
TABLE 11
ALLOWABLE WITHDRAWAL LOADS FOR LAG SCREWS

<table>
<thead>
<tr>
<th>Units in pounds per inch of thread penetration</th>
</tr>
</thead>
</table>

*Source: Wood Technology in the Design of Structures*
TABLE 12
ALLOWABLE LATERAL LOADS FOR LAG SCREWS

Source: Wood Technology in the Design of Structures
TABLE 13
ALLOWABLE WITHDRAWAL AND LATERAL LOADS FOR WOOD SCREWS

Units are in pounds per inch of thread penetration

Source: Wood Technology in the Design of Structures
TABLE 14
ALLOWABLE LOADS ON ONE BOLT IN DOUBLE SHEAR

*Source: Wood Technology in the Design of Structures*